The Choice of Estimation Method and Its Effect on Efficiency Measurement in Public Education: Stochastic Frontier Regression vs. Data Envelopment Analysis

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ABSTRACT. The purpose of this paper is to extend the empirical literature on efficiency measurement in public education. Two estimation methods often used in determining efficiency in the production of public education are used to determine if the results from the methods are similar. The Oklahoma public schools are of interest because Oklahoma has a large number of districts with very different characteristics. The methods of estimation are Stochastic Frontier Regression (SFR) and Data Envelopment Analysis (DEA). SFR estimates the inefficiency model simultaneously with the production or cost function. In the DEA model, the first stage estimates the efficiency scores and the second stage uses a Tobit regression model to determine causes of inefficiency. In this study, the empirical results of the SFR and DEA efficiency scores for the majority of Oklahoma school districts are not identical, suggesting that the method of estimation affects the efficiency scores. In general, SFR generated a more favorable score than that of DEA. The results from the two estimation methods in the inefficiency model are also different. However, both methods suggest that the most important determinants of inefficiency are socioeconomic factors associated with each district. (I21, C13)

I. Introduction

The operational funding for public schools in the United States comes directly from tax revenue, and consequently taxpayers expect public schools to attain a certain level of quality in the provision of educational services. While schools' real expenditures have been increasing, standardized test scores—often used indicators of school quality—have shown little if any improvement (US Department of Commerce 1999, Tables 253,254 and 296).

In their book, *Assessing Education Practices,* Baumol and Becker (1995) state that real expenditures on secondary and elementary education

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more than tripled between 1960 and 1990, resulting in a lower student-teacher ratio and a rise in the average age, education, and specialization level of teachers. However, performance continues to decline, leading them to refer to educational expenditures as deceptive indicators. Serious questions have been raised about the management and efficiency of public schools. The U.S. Efficiency studies suggest a different question about the link between school funding and performance: “Can schools reallocate existing expenditures in ways that improve performance?”

A major educational reform law, House Bill 1017, was passed in early 1990 in an attempt to improve the quality of education in Oklahoma. Because the funding came from a tax increase as well as reallocation of state funds toward common education and away from other popular programs, the law created much controversy concerning its effectiveness.

The literature has approached the efficiency measurements using two broad methodologies, Parametric-Stochastic Frontier Regression (SFR), and Nonparametric-Data Envelopment Analysis (DEA). Coelli et al. (1998) offer a detailed explanation of these methods. Both approaches have inherent limitations and the recent literature recognizes the complementary nature of the two methods.[1]

Public education efficiency studies have generally used a single output measure with single period (cross-section) or multi-period (panel) data (e.g., Chakraborty et al. 2001, Adkins and Moomaw 1997). The principal advantage of panel data is that, for each school district, the data are observed over several time periods and this may reduce the bias due to the effect of omitted variables on the estimated relationship. However, according to Wyckoff and Levinge (1991), the estimate of efficiency is sensitive to the selection of output as well and therefore the use of a single output to measure these efficiencies is somewhat restrictive.

In an attempt to extend previous efficiency studies in public education, this study uses data for three academic years (1996-1999) from Oklahoma school districts. To estimate the technical efficiency for each district and its determinants, multiple outputs and two types of inputs are employed. The efficiencies are estimated using both SFR and DEA methods and then, compared.

II. Review of Literature

A production function is a mathematical expression that relates inputs to outputs for a given technology. The production function indicates the maximum output attainable with a given vector of inputs. For a costminimizing firm the goal is to produce a certain level of output at minimum cost, given input prices.

In the context of efficiency measurement, the literature tends to refer to the production function as the production frontier to stress the maximal property of the function (Coelli et al.1998, p.12). Firms operating on the frontier (production or cost) are said to be technically efficient. If a firm operates below its production frontier or above its cost frontier, then the firm is technically inefficient.

Before Farrell's (1957) introduction of the frontier approach in estimating the dual functions' efficiency, the linear average mean response functions (production or cost) were estimated using least squares (OLS) or some variant thereof. The average functions do not necessarily represent the best technology,[2] and therefore, there is an explicit conceptual link missing between microeconomic definitions of production or cost functions and what is being estimated. A frontier function is a bounding function against which inefficiency or the relative size of one-sided deviations from the maximum output or the minimum cost can be estimated. Farrell offers a measure of technical efficiency (TE), which equals one minus the maximum equal-proportional reduction of the input vector, given an output vector. Then, TE can take values between 0 and 1, and hence the degree of inefficiency of the firm can be measured.

Aigner and Chu (1968) estimated a Cobb-Douglas parametric frontier production function, using data on a sample of N firms. In their model the disturbance term measures technical inefficiency due to factors that are under the firm's control, i.e., a deterministic model. The shortcoming of the deterministic model is that it does not account for other sources of disturbance noise, such as measurement error, luck, climate, etc., which are not under a firm's control and can be favorable or unfavorable to the firm's output. To address this shortcoming, Aigner et al. (1977) suggested a new model where output is bounded above by a stochastic frontier, which accounts for noise. The stochastic frontier models suffer from the common criticism that there is no a priori justification for selection of any particular distributional form for the deterministic disturbance term (Coelli et al.1998). However, there is no evidence that the choice of distributional assumptions have a significant effect on predicted technical efficiencies (C.A.K Lovell 1995 in Coelli et al. 1998 p.187).

A number of earlier studies applied two-stage least squares estimation (e.g., Pitt and Lee 1981, Kalirajan et al.1983 and Kalirajan 1989).[3] Kumbhakar et al. (1991) suggest at least two problems with the two stage method. First, inefficiencies and input may be correlated; if so the estimated inefficiencies as well as the parameters of the second stage are inconsistent. Second, the use of OLS in the second stage ignores the fact that the dependent variable is inherently one sided, i.e., inefficiencies cannot take on negative values. The authors developed a single-step Maximum Likelihood Estimation (MLE) technique and claimed that this technique avoids the usual criticism targeted at the two-stage procedure.[4]

A parallel approach to the econometric estimation of stochastic frontier models is the mathematical programming approach. Charnes, Cooper and Rhodes (1978) pioneered the concept of data envelopment analysis (DEA), which involves the use of linear programming. One of the serious shortcomings of the DEA approach is that the sampling distributions of DEA estimators are unknown.

Kirjavainer and Loikkaner (1998) studied the efficiency of Finnish senior secondary schools with DEA. The authors acknowledge that in addition to the lack of identifiable statistical properties, the DEA model does not account for one of the most significant and robust results of schools' input-output studies, the effect of the socioeconomic factors not under the control of the school. Following McCarty and Yaisawarng (1993), Kirjavainer and Loikkaner (1998) suggest the use of a two-stage model. The first stage uses DEA to calculate the efficiency scores using variables that are controlled by schools. The second stage involves the use of ML estimation of the Tobit regression model, which has well known and desirable statistical properties. The Tobit model provides efficiency measures based on variables that are not included in the DEA and are possibly outside of the decision-making power of schools.

III. Educational Production

Many economic studies of educational production, efficiency, and cost structure have been inspired by the Coleman report (Coleman et al.1966). In analyzing production frontier models in education, the use of the production function rather than the cost function maybe more practical since data on input prices generally are not available. The Coleman report suggested an input-output relationship between administrative resource allocation and students' achievements. Thus, the production process of education consists of school and non-school inputs, which produce multiple outputs (achievement test scores). The report also suggests that students' performance was related largely to their socioeconomic background rather than the variation in schools (Hanushek 1986, 1989).

Subsequent analyses of school performance differ in their focus and methodology. However, Hanushek's 1986 survey of 147 studies suggests that most studies include expenditures per pupil, student/teacher ratio, teacher education and experience as well as family characteristics as the primary determinants of student achievement. These studies consistently conclude that (i) expenditure and student performance are not systematically related and (ii) family characteristics have an effect on the students' achievement.

The possible effect of school size and/or district size on student performance cited by researchers captures the possible effect of economies of scale on schools' productivity. The results from studies of the effects of the scale economies associated with public schools are inconsistent; some find evidence of economies of scale and others do not.

Barrow (1991), and Wyckoff and Lavinge (1991) found that efficiency measures are sensitive to the estimation method as well as the definition of educational output. Hanushek (1979, 1986) suggests that measuring and defining educational inputs and outputs can be problematical and data availability may necessitate compromise regarding model selection. In addition, consideration must be given to the definition of efficiency and how it is being measured.

For the most part, Oklahoma public education studies agree with the national studies. For example, Adkins and Moomaw (1997) found that the relationship of test scores with respect to spending is positive but very small. Their findings also suggest that districts with more experienced teachers are generally more efficient and that districts that pay higher salaries get better results.

In addition Abdul Rahman (1996) and Adkins and Moomaw (1997) suggest the existence of economies of scale in Oklahoma public schools. That is, larger districts in Oklahoma tend to be more efficient than smaller districts. Thus districts might benefit by consolidation. And finally, Metzger and Barta (1999) suggest that the likely causes of the contradictory conclusions regarding expenditures and district structure are errors in data and model choice, as well as model specification errors (econometric issues).

IV. The Data Set

The data for the present study were obtained from the Oklahoma Department of Education, Office of Accountability for the academic years 1996-1997, 1997-1998, and 1998-1999. The data includes observations on several socioeconomic indicators (e.g., students eligible for the subsidized lunch program, parents' education level, family income, etc.). Students' performance measures are based on different standardized test scores appropriate for the different grades (e.g., ITBS, CRT, ACT scores) for over 600 school districts in Oklahoma. In order to maintain a balanced panel in this study, dependent schools, which do not offer grades K-12 and those districts with incomplete data, have been eliminated from the sample. The resulting sample includes observations on 366 school districts.

Among the numerous measures of performance available, the Iowa Test of Basic Skills (ITBS) and Criterion Reference Test (CRT) are probably most reliable. Hanushek (1986) acknowledges that test scores are imperfect measures of educational output. However, test performance is used to allocate funds and evaluate educational programs. Test scores are also commonly available and appear to be valued by educators, as well as parents and decision makers as a measure of education efficacy. Here, ITBS for grades 3 (IT3) and 7 (IT7) and CRT for grades 8 (CRT8) and 11 (CRT11) are used as measures of educational output. Summary statistics of the variables (and their definitions) for the panel can be found in Table 1.

V. Two Equation Stochastic Frontier Regression (SFR)

Following Battese and Coelli's (1995) theoretical model, the stochastic production function of Oklahoma schools for the periods of 1996-1999 is modeled with a translog functional form because of its flexibility:

$$\begin{aligned} lnScore_{it} = {} & \beta_0 + ln(I_{it})\beta_1 + ln(O_{it})\beta_2 + [ln(I_{it})]^2\beta_3 \\ & + [ln(O_{it})]^2\beta_4 + ln(I_{it})ln(O_{it})\beta_5 + (V_{it} - U_{it}) \end{aligned} \tag{1}$$

where output ($Score_{it}$) is a measure of districts' performance on one of the several standardized tests for district *i* in time period *t*: IT3, IT7, CRT8 and CRT11. Inputs are I and O which are under the control of school districts. The variables V_{it} are the random disturbance terms, i.e., standard white-noise disturbance, and U_{it} are non-negative random variables associated with technical inefficiencies. V_{it} and U_{it} are assumed to be independent, with covariance of zero.

TABLE 1–Summary Statistics for Variables Used in Production Analysis of Oklahoma Schools (1996-1999)

	Mean	Maximum	Minimum	Std. Dev.
IT3	62.2377	93	26	9.949506
IT7	55.6949	85	30	8.192466
CRT8	74.18628	98.6	36.66667	9.39291
CRT11	69.06264	94.25	29	9.80891
O	1992.386	5468.443	1143.357	463.7583
I	2859.53	5767.599	1961.999	498.4934
ADM	1555.872	41471.46	143.91	3767.604
SALARY	29615	35334.72	26608	1190.961
DEG	32.49643	80.62	3.69	13.33091
YRSEXP	15.30133	30.6667	5.830112	4.562163
LUNCH	51.18705	100	4.477241	15.93732
MIN	27.36814	100	0	16.58325
STR	15.95792	21.97037	8.162783	2.200199

IT3: ITBS for grade 3 (composite scores)
IT7: ITBS for grade 7 (composite scores)
CRT8: CRT for grade 8 (average scores)
CRT11: CRT for grade 11 (average scores)
I: Instructional expenditure per student ($)
O: Non-instructional expenditure per student ($) i.e., administrative and other expenses that are not directly used for instructional purposes.
ADM: Average daily membership (number of students)
SALARY: Average salary per full-time equivalent teacher ($)
DEG: Percentage of teachers with advanced degree
YRSEXP: Average experience of teachers (year)
LUNCH: Percentage of students eligible for reduced cost or free lunch
MIN: Percentage of minority students
STR: Student/teacher ratio

Following Pitt and Lee (1981), incorporating environmental variables in equation (1), the model can be rewritten as:

$$\begin{aligned} ln\,Score_{it} = {} & \beta_0 + MIN_{it}\beta_1 + LUNCH_{it}\beta_2 + ln(I_{it})\beta_3 + ln(O_{it})\beta_4 + [ln(I_{it})]^2\beta_5 \\ & + [ln(O_{it})]^2\beta_6 + ln(I_{it})ln(O_{it})\beta_7 + (V_{it} - U_{it}) \end{aligned} \tag{2}$$

Equation (2) assumes that any change in environmental variables causes a parallel shift in the frontier, i.e., a change in the intercept. The inefficiency effect, U_{it} in equation (2), is modeled as a function of several variables:

$$\begin{aligned} U_{it} = {} & \delta_0 + \delta_1 SALARY_{it} + \delta_2 YRSEXP_{it} + \delta_3 DEG_{it} + \delta_4 ADM_{it} + \delta_5 ADM_{it}^2 \\ & + \delta_6 STR_{it} + e_{it} \end{aligned} \tag{3}$$

SALARY, YRSEXP and DEG represent input quality and are under the control of school districts. ADM, ADM^2 and STR measure the different quantity adjustments by the districts. The β,δ coefficients along with the variance parameters which are expressed as:

$$\sigma^2 = \sigma_U^2 + \sigma_V^2$$

$$\gamma = \frac{\sigma_U^2}{\sigma_U^2 + \sigma_V^2}$$

are unknown parameters to be estimated.

VI. Data Envelopment Analysis and Second-Stage Tobit Model (DEA)

In this section, the two-stage specification of the non-parametric model data envelopment analysis in the first stage and Tobit regression in the second stage is discussed. In DEA, it is assumed that all firms have the same deterministic production frontier and any deviation from the frontier is due to inefficiency. The basic idea of this approach is to view schools as productive units with multiple inputs and outputs.

To measure technical efficiency, output-oriented DEA uses the same inputs and outputs as the stochastic frontier model. This method identifies the technical efficiency as a proportional increase in the output vector with a given input vector. Therefore, for the production frontier in equation (2), the output-oriented measure of technical efficiency is the solution to the following constant returns to scale (CRS) DEA linear programming problem (Coelli et al. 1998):

$$\begin{aligned} &\max_{\phi,\lambda} \phi \\ &s.t. \quad -\phi y_i + Y\lambda \geq 0 \\ &\qquad\ \ x_i - X\lambda \geq 0 \\ &\qquad\ \ \lambda \geq 0 \end{aligned} \tag{4}$$

where ϕ is a scalar, y_i and x_i are column vector of outputs (IT3, IT7, CRT8, CRT11) and column vector of inputs (MIN, LUNCH, I, O) for the ith school district respectively. λ is an N x 1 vector of constants. The variable Y is an M x N output matrix (M = 4) while X is a K x N input matrix (K = 4). The proportional increase in outputs that could be achieved by the ith school district, holding inputs constant, is ϕ-1 where $1 \leq \phi < \infty$. In addition $1/\phi$ is the school district's efficiency score and is between 0 and 1.

To assess the effects of variables not included in the first stage on technical efficiency, McCarty et al. (1993) suggest using efficiencies generated by DEA as dependent variables in a Tobit regression:

$$\begin{aligned} Y_{it}^T = {} & \beta_0^T + \beta_1^T \text{YRSEXP}_{it} + \beta_2^T \text{DEG}_{it} + \beta_3^T \text{SALARY}_{it} \\ & + \beta_4^T \text{ADM}_{it} + \beta_5^T \text{ADM}_{it}^2 + \beta_6^T \text{STR}_{it} + e_{it} \end{aligned} \tag{5}$$

where the T superscript denotes Tobit and the $Y_{it}^T s$ are the DEA efficiency estimates. Since the efficiency estimates from the first stage are between 0 and 1, data is censored. Thus, Tobit regression, as opposed to OLS, is the appropriate method of estimation for equation (5). The explanatory variables in equation (5) are the variables of the technical inefficiency equation of the stochastic frontier model (equation 3). The possibility of the existence of heteroscedasticity in this stage should be considered. If in fact it exists, it should be incorporated into the model in order to have efficient parameter estimates.

VII. Empirical Results

The elasticities of output with respect to instructional (I) and noninstructional (O) expenditures based on the parameter estimates of the OLS model from the panel data, for each grade are presented in Table 2. These elasticities are computed using the sample mean values of ln (I) and ln (O).

TABLE 2–Elasticities of Test Scores with Respect to Instructional and Noninstructional Expenditures Evaluated at their Asymptotic Means

	IT3	**IT7**	**CRT8**	**CRT11**
I	.197	.174	.159	.0918
Standard Error	.022	.016	.02	.019
O	.064	.123	.0515	.0683
Standard Error	.032	.0158	.0326	.008

The elasticities of test scores with respect to expenditures are positive and statistically significant for all grades at the 5 percent level. With respect to instructional expenses, the estimated effect is greater in grade 3 than in the other grades considered. However, the elasticities are fairly small even in grade 3. This result is consistent with findings of Adkins and Moomaw (1997). A one percent increase in instructional spending is expected to increase the 3rd grade ITBS scores by almost .2 percent, .17 percent for grade 7, .16 percent for grade 8, and .09 percent for grade 11.

Smaller elasticities with regard to noninstructional expenses, ranging from .05 to .12 suggest that reallocation of noninstructional spending to instructional spending may result in a small improvement of test scores for all grades. This result is consistent with the findings of Adkins and Moomaw (1997) except for grade 7.

To get some preliminary results, the SFR model for each grade was estimated using FRONTIER 4.1 developed by Coelli (1994). The null hypothesis that the inefficiency effects are not a linear function of the right hand side variables in equation (3) was tested using the likelihood ratio test and rejected for all grades at the 5 percent level. This suggests that the joint effect of the explanatory variables on the technical efficiencies is significant. However, these findings are conditional on the homoscedasticity of the model. If the data are heteroscedastic, as the results from White's tests suggest, then inferences may not be statistically valid.

To determine the degree of robustness of this model, a DEA is performed that permits some flexibility in this specification. In addition, DEA allows more suitable modeling of the district level production function since it allows for multiple outputs.

The first stage output-oriented DEA model:

$$\text{Scores}_{it} = f\,(\text{MIN, LUNCH, I, O})$$

includes the same outputs and inputs as the SFR model. The SFR model estimation suggests that MIN and LUNCH have a negative effect on the test scores; since the directions of the effects are known, these variables can be directly included into the linear programming problem (Coelli et al.1998). Since output-oriented DEA is a maximization problem, the complement of MIN and LUNCH instead of the variables themselves are considered:

$$\text{Scores}_{it} = f\,(\text{MIN*, LUNCH*, I, O}) \tag{6}$$

where Scores_{it} is IT3, IT7, CRT8, CRT11. The percentage of non-minority students (1-MIN) is MIN*, LUNCH*is the percentage of students not eligible for subsidized or reduced LUNCH (1-LUNCH) and I, O are expenditures.

Equation (6) is estimated using DEAP (2.1) software developed by Coelli (1996). Table 3 presents basic information on the distribution of efficiency scores generated by the DEA model under both constant returns to scale (CRS) and variable returns to scale (VRS) assumptions for 1996-1999.

Interestingly, under both CRS and VRS assumptions, the average efficiency scores for the sample has declined and the variation has increased every year, suggesting that school districts actually became less efficient with more variation in the level of efficiency among districts throughout the 1996-1999 academic years. Summary statistics of the DEA efficiency scores under both CRS and VRS for each year and the panel are presented in Table 3.

TABLE 3–Summary Statistics for DEA Efficiency Scores

	1996-1997		1997-1998		1998-1999		Panel	
	CRS	VRS	CRS	VRS	CRS	VRS	CRS	VRS
Mean	.8918	.9256	.8743	.9102	.8557	.8837	.8706	.9065
SD	.8486	.672	.9287	.7165	.8956	.7279	.9108	.7262
Minimum	.581	.673	.529	.647	.561	.605	.529	.605
Maximum	1	1	1	1	1	1	1	1

Since VRS could bias the efficiency scores upward (Coelli et al.1998), the CRS efficiency scores are more appropriate as the independent variable in the second stage. Tobit regressions are computed using the LIMDEP 7.0 software, which corrects for the existence of heteroscedasticity in the model. All the explanatory variables in equation (4) as well as the independent variable, CRS efficiency, are considered as the possible source of this misspecification. However, CRS efficiency scores, student/teacher ratio and the size of the school districts as measured by ADM are likely sources of heteroscedasticity. Thus, a heteroscedastic Tobit regression with these variables as sources of heteroscedasticity is computed.

To test the heteroscedasticity hypothesis, a generalized likelihood ratio test was employed. The test statistic, $\lambda = -2[\log(\text{likelihood } H_0) - \log(\text{likelihood } H_1)] \sim \chi^2_J$ where J equals the number of variables considered as a potential source of heteroscedasticity, suggests that the homoscedasticity hypothesis should be rejected. Therefore, there is substantial evidence that at least one of the three variables above explains the existence of heteroscedasticity in the Tobit regression.

The Tobit coefficient estimates, computed under the assumptions of homoscedastic and heteroscedastic error terms in the model, are presented in Table 4. The results are generally similar under the two assumptions which perhaps suggest that the biases created by heteroscedasticity are not very large. In both models, DEG, YRSEXP, and STR have a statistically significant positive effect on efficiency and the effect of SALARY, ADM, and ADM^2 are all negligible.

TABLE 4–Tobit Coefficient Estimates of the Efficiency Model
Dependent Variable: Efficiency Estimates from the First-Stage
DEA CRS Model

	Homoscedastic		Heteroscedastic	
Variable	Coefficient	t-statistic	Coefficient	t-statistic
Constant	.666951	7.584	.726393	8.830
SALARY	-.000005	-1.829	.099482	-1.433
DEG	.089591	4.091*	.099482	4.580*
YRSEXP	.003067	4.126*	.002656	3.900*
STR	.018588	12.985*	.013556	9.057*
ADM	-.000035	-.174	.000008	.048
ADM^2	-.000000	-.160	-.000000	-.488

* Significant at the .05 level

VIII. SFR vs. DEA

In order to compare the Stochastic Frontier Regression (SFR) and the Data Envelopment Analysis (DEA) models, the data set on Oklahoma school districts is used in several different ways to facilitate the comparison between the results from these models.To see whether or not a different functional form would affect the computation of the efficiency scores for the SFR model, a Cobb-Douglas SFR is estimated. The efficiency scores for SFR under Translog and Cobb-Douglas specification are almost identical. Therefore, subsequent analysis is based on the results of SFR with Translog specification.

To begin with, the four output categories are used directly in the DEA model. However, since SFR does not allow for multiple outputs, in the past, researchers faced with this situation estimated the production function using a single aggregate output measure (Coelli et al.1998). Thus, for comparison purposes, the dependent variable in SFR is computed as the logarithm of the average of all the test scores in the panel. The results of the SFR model are presented in Table 5.

TABLE 5–Maximum-Likelihood Estimates for Parameters of Stochastic Production Frontier and Inefficiency Models for Oklahoma School Districts (Panel Data Model)
Dependent Variable: ln (Average of all Test Scores)

Variable	Parameter	Coefficient	Standard Error	t-ratio
Stochastic Production Frontier				
Constant	β_0	-12.69181	1.15505	-10.98811
MIN	β_1	-0.14036	0.01719	-8.16405
LUNCH	β_2	-0.30118	0.01928	-15.62106
1n(I)	β_3	1.95900	0.66211	2.95871
1n(O)	β_4	2.24843	0.65266	3.44504
$[1n\ (I)]^2$	β_5	0.15694	0.08202	1.91335
$[1n\ (0)^2$	β_6	0.15582	0.05031	3.09743
1n(I)1n(O)	β_7	-0.57053	0.11323	-5.03870
Inefficiency Equation				
Constant	δ_0	0.55581	0.17862	3.11170
*SALARY	δ_1	-0.20812	0.06965	-2.98807
YRSEXP	δ_2	-0.45472	0.08305	-5.47490
DEG	δ_3	0.00148	0.00214	0.69187
*ADM	δ_4	-0.00908	0.00240	-3.78644
$*ADM^2$	δ_5	0.00223	0.00058	3.86501
STR	δ_6	0.01119	0.00448	2.49717
Variance Parameters	$\sigma^2 = \sigma_U^2 + \sigma_V^2$	0.16949	0.00247	6.85311
	$\gamma = \frac{\sigma_U^2}{\sigma_U^2 + \sigma_V^2}$	0.84875	0.2975	28.52959
Log Likelihood Function		127.39500		
LR test of the one-sided error				
(H_0: $\gamma = 0$)	187.34200			
number of restrictions		8		

*Variable is scaled:
SALARY is SALARY/1,000
ADM is ADM/100
$ADM^2$is ADM^2/1,000,000

Furthermore, the SFR and DEA CRS models are estimated for each grade and each year of the 3-year period under study. In order to examine the effect of technological changes over the years under study, the panel data for each grade is used with both the SFR and DEA models. Therefore, 12 different cross-section (N = 366) and 4 panel (N = 1098) SFR and DEA CRS models are estimated to obtain the efficiency scores under each model. To obtain the correlation between the results of the two models, school districts are ranked based on their efficiency scores and Spearman rank correlations between these rankings are computed. The results are presented in Table 6.

TABLE 6–Spearman Rank Correlation Coefficients of DEA CRS and SFR Efficiency Scores

Grade	1996-1997	1997-1998	1998-1999	Panel
3	.82	.85	.77	.80
7	.79	.86	.82	.82
8	.72	.70	.79	.76
11	.87	.81	.80	.80

These results suggest that the efficiency rankings generated by both models are generally similar for the cross-sections as well as the panels, suggesting that sample size and technological changes do not seem to have a strong influence on the efficiency ranking. However, when multiple output DEA and SFR are considered for the panel their Spearman rank correlation is computed to be .6. This could suggest that in the presence of multiple outputs, SFR is not a very reliable estimation technique.

To compare and examine the effects of the explanatory variables in equations (3) and (5) on the efficiency, the parameter estimates of SFR and heteroscedastic Tobit are compared. Note that in SFR, equation (3) has inefficiency as the dependent variable, while in the DEA second-stage Tobit regression that is in equation (5) the dependent variable is efficiency. Thus, in order to simplify the comparison between the two models, the signs of the SFR parameter estimates are inverted. The results for both models are presented in Table 7.

According to the results in Table 7, the DEA and SFR models suggest that teachers' years of experience and the size of the school districts measured by ADM have a positive effect on efficiency. However, the effect of ADM in the DEA model is not statistically significant. As for the effect of teachers' salary on efficiency, SFR suggests a positive and significant effect vs. DEA, which suggests the opposite.

Teachers holding advanced degrees, according to the DEA model, affect efficiency positively and the corresponding coefficient is statistically significant whereas SFR suggests just the opposite. With respect to the student/teacher ratio, the effect on efficiency is positive and significant in the DEA model and negative and significant in the SFR model.

TABLE 7–Parameter Estimates; Dependent Variable: Efficiency Estimates

	SFR		**DEA-Tobit Heteroscedastic**	
	Coefficient	**t-ratio**	**Coefficient**	**t-ratio**
Constant	-.5558	-3.1117	.72639	8.830
SALARY	.2081	2.988*	-.000004	-1.433
YRSEXP	.4547	5.4749*	.002656	3.900*
DEG	-.0014	-.69187	.099482	4.580*
ADM	.0090	3.7864*	.000008	.048
ADM^2	-.0022	-3.865*	-.000080	-.488
STR	-.01119	-2.497*	.013556	9.057*

*Significant at the .05 level.

One possible reason for a lower correlation between the multiple output DEA and SFR models, as well as differences in Table 7, could be the fact that SFR simply does not allow for multiple outputs. Thus, taking the average of all outputs to overcome this restriction may not be appropriate. Therefore, in situations where multiple output rather than single output assumption is more realistic, such as the case of Oklahoma public schools, DEA models with second-stage Tobit regression may be more reliable in explaining the efficiency differences among school districts.

IX. Conclusion

This study uses two methods to estimate efficiency in the production of education in light of possible empirical specification problems caused by possible structural changes in data collection and by the possible heteroscedasticity in the error term. A review of production frontier studies suggests the use of different methods of estimation based on the available data and model specification.

The Stochastic Frontier Regression (SFR) and Data Envelopment Analysis (DEA) are the estimators considered in this study and the results are compared to see how robust these methods are. The existing literature includes a relatively small number of applications of the stochastic production frontier approach to school districts, none of which considers the existence of heteroscedasticity.

The data set includes observations on various input and output measures (e.g., teacher salary, the size of the district etc.) for 366 independent (K-12) school districts in the state of Oklahoma. The time period under consideration consists of a three-year period that encompasses the 1996-97 through the 1998-99 academic school years.

The results suggest that there are varying degrees of technical inefficiency among Oklahoma school districts. Therefore, the average response function (OLS) cannot adequately represent the production function of Oklahoma school districts. Thus a two-equation stochastic production frontier model is a preferred alternative model. This conclusion is supported by hypothesis tests which indicate that inefficiency effects have both systematic and random components.

The existence of heteroscedasticity in the data is also supported based on hypothesis tests. However, to determine the extent of heteroscedasticity, a model assuming homoscedastic error is estimated. The estimation results based on the homoscedastic Stochastic Frontier model suggest that the signs of the coefficients of the explanatory variables are in general as expected but that these estimates may not be robust in the presence of heteroscedasticity.

In addition to the problem of heteroscedasticity, since the model consists of multiple outputs, the existing literature suggests the use of distance-functions, which allows for multiple output, rather than stochastic frontier functions. Thus, the non-parametric approach to the estimation of efficiency was employed, i.e., the DEA approach.

DEA suffers from a lack of well-known statistical properties and is not therefore very useful in answering questions regarding whether money matters. In addition, the production function is not parameterized and it yields no estimates of the various spending elasticities. To overcome these shortcomings a second-stage Tobit regression, which has well known statistical properties, was employed to explain the effects of variables such as teacher salary (SALARY), teacher years of experience (YRSEXP), teachers holding advanced degree (DEG), size of school district (ADM), and student/teacher ratio (STR) on the efficiency scores generated by the DEA model. Tobit regression is appropriate since the efficiency scores (dependent variable) are between 0 and 1.

Heteroscedasticty is accounted for in the Tobit regression and thus, the efficient coefficient estimates of the variables are found to be, in general, consistent with the expected hypothesis. However, with the exception of two variables, teachers holding advanced degrees and the student/teacher ratio, the effects of other variables on efficiency measures are not significantly different from zero.

The results from the potentially misspecified homoscedastic stochastic production frontier are compared to the heteroscedastic Tobit model estimated from DEA efficiencies. The results are fairly similar suggesting that perhaps the biases created by heteroscedasticity are not very large. In view of the above considerations, the estimates obtained in this study are more reliable than those of past studies, which were generally based on the average response function, cross-section data and/or the single output assumption.

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Endnotes

1. See the Supplemental Issue in the *Journal of Econometrics* 1990 for a review.
2. The average function assumes that all firms are efficient, which is generally not a reasonable assumption.
3. For a discussion of the two stage OLS and the problems associated with it see Green (1980) and Kumbhakar et al. (1991).
4. Many researchers such as Reifschneider and Stevenson (1991), Huang and Liu (1994), Coelli (1995a), and Battese and Coelli (1995) have also employed MLEs, preferring them to the two-stage methods.