

Myopic Firm Behavior Thwarting Intent of Average-Revenue-Lagged Regulation

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ABSTRACT. Recent papers have demonstrated that Average-Revenue-Lagged regulation (ARL) as applied to the single-product / multi-market firm is susceptible to strategic manipulation even leading to global profit maximum pricing. This paper illustrates that myopic behavior within ARL under appropriate conditions on product demands and production costs can also produce profit maximum prices thus undermining obvious regulatory intent. The possibility of this outcome is another significant criticism that may be leveled against this particular regulatory approach. (L43, L51)

I. Introduction

Regulatory methods in the United States as applied to public utilities were once uniformly of the rate-of-return type until economists of the mid-to-late twentieth century began to publicize shortcomings with this approach. Averch and Johnson (1962) demonstrate that rate-of-return regulation creates perverse incentives for the firm to utilize its resources in inefficient ways. Consequently, since the early 1980s much study has been devoted to alternative price-incentive arrangements that utilize a price cap designed to encourage implementation of cost-saving production methods and technologies. Typically, the resulting savings are eventually transferred to the consumer as the price cap is adjusted downward after a predetermined number of regulatory periods (Acton and Vogelsang, 1989; Braeutigam and Panzar, 1993; Vogelsang, 2002).

For single-product / multi-market firms, Average-Revenue-Lagged regulation (ARL) is one such price-incentive method that is attractive because it induces the myopic firm to move over time to efficient prices.¹ Further, as only lagged demand quantities are employed in determining average prices, there is no need for demand forecasting as is required under the standard Average Revenue (AR) approach. However, enthusiasm for ARL based on these traits must be tempered by Cowan's

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(1997) demonstration that steady-state consumer surplus under ARL is lower than that existing at initial prices.

Additionally, Sappington and Sibley (1992), and Foreman (1995) illustrate how strategic manipulation of revenue weights can increase total profits at consumer expense whenever intertemporal linkages of the form existing under ARL regulation are present.² Moreover, Currier (2005) verifies that strategic price manipulation under certain realistic properties for the product demand functions can enable the firm to achieve global profit maximum prices, thus rendering the regulatory constraint ineffective. This paper illustrates the possibility of a similar result obtaining under myopic behavior given linear product demands and constant marginal production costs. As such, the argument herein presents perhaps a more damaging indictment against the wisdom of employing ARL regulation toward the goal of protecting consumer surplus.

II. Preliminary Definitions and Relationships

Consider ARL as applied to the single-product firm that sells its product in n markets. Note that consumer surplus corresponding to the n -dimensional price vector $\mathbf{p} = (p_1, p_2, \dots, p_n)$ where linear demand functions $q_i = a_i p_i + b_i$ prevail in each market $i = 1, 2, \dots, n$ is given by:

$$V(\mathbf{p}) = \sum_{i=1}^n -\frac{1}{2} \left(a_i p_i^2 + 2b_i p_i + \frac{b_i^2}{a_i} \right) = \sum_{i=1}^n -\frac{a_i}{2} \left(p_i + \frac{b_i}{a_i} \right)^2 \quad (1)$$

which is C^2 (i.e. twice continuously differentiable) and convex since the

Hessian matrix of $V(\mathbf{p})$, which is $H(\mathbf{p}) = \begin{bmatrix} -a_1 & 0 & \dots & 0 \\ 0 & -a_2 & \dots & 0 \\ 0 & \vdots & \ddots & 0 \\ 0 & 0 & \dots & -a_n \end{bmatrix}$, is

positive definite. (We are assuming zero cross-price elasticities of demands between markets).³ For $n = 2$ the consumer surplus function is a convex paraboloid having domain $\{(p_1, p_2) | 0 \leq p_i \leq -b_i/a_i, i = 1, 2\}$ in the p_1, p_2 -plane and possessing a homothetic family of elliptically-shaped iso-surplus curves centered at $(-b_1/a_1, -b_2/a_2)$. Homotheticity

guarantees that this family of curves possesses a constant slope along any ray emanating from the family center (an important property that we will make use of later). Additionally, total differentiation of (1) for $n = 2$ gives the slope of any iso-surplus as:

$$\frac{dp_2}{dp_1} = \frac{a_1 p_1 + b_1}{a_2 p_2 + b_2} = - \frac{q_1(p_1)}{q_2(p_2)}. \quad (2)$$

The firm's profit function under constant marginal costs of production $\mathbf{c} = (c_1, c_2, \dots, c_n)$ is $\pi(\mathbf{p}) = (\mathbf{p} - \mathbf{c}) \cdot \mathbf{q}(\mathbf{p}) - F$ where $\mathbf{q}(\mathbf{p})$ is the vector of n market demand quantities corresponding to the n -dimensional price vector $\mathbf{p}$.⁴ For $n = 2$ markets the profit function takes the form:

$$\pi(\mathbf{p}) = -F + \sum_{i=1}^2 (p_i - c_i)q_i(p_i) = -F + \sum_{i=1}^2 (p_i - c_i)(a_i p_i + b_i). \quad (3)$$

This function is C^2 , concave, and maximized at some unique global profit maximum price vector $\mathbf{p}^*$. Expanding and completing the square of (3) in p_1 and p_2 allows us to rewrite the iso-profit locus equation for any feasible π in the standard form:

$$\pi = -F + \sum_{i=1}^2 a_i \left[p_i + \frac{(b_i - a_i c_i)}{2a_i} \right]^2 - \frac{(b_i + a_i c_i)^2}{4a_i} \quad (4)$$

which represents an elliptically-shaped curve with center $\left(-\frac{(b_1 - a_1 c_1)}{2a_1}, -\frac{(b_2 - a_2 c_2)}{2a_2} \right)$. This derivation not only has given us one approach to finding the global profit maximum:

$$\mathbf{p}^* = \left(-\frac{(b_1 - a_1 c_1)}{2a_1}, -\frac{(b_2 - a_2 c_2)}{2a_2} \right), \quad (5)$$

but also implies an important result; the family of iso-profit curves under linear demands and constant marginal costs is homothetic and elliptically shaped. Finally, the slope of any iso-profit in the p_1, p_2 -plane is:

$$\frac{dp_2}{dp_1} = - \frac{2a_1p_1 + b_1 - a_1c_1}{2a_2p_2 + b_2 - a_2c_2} \quad (6)$$

Equating the expression for the iso-profit slope with that for the iso-surplus slope yields a linear equation defining the set E of efficient price vectors:

$$E = \{(p_1, p_2) \mid a_1(a_2c_2 + b_2)p_1 - a_2(a_1c_1 + b_1)p_2 + a_2b_1c_2 - a_1b_2c_1 = 0\}. \quad (7)$$

Note that E connects the center of the iso-profit family with the center of the iso-surplus family. Finally, we define Δ^+ to be the set of equal prices, which in the p_1, p_2 -plane is the 45° line extending upward and rightward from the origin. See Figure 1.

Figure 1. Iso-surplus and iso-profit curves with set E (efficient prices)

III. The Mechanics of Average-Revenue-Lagged Regulation

During a typical period t of ARL regulation the firm is allowed to adjust its prices to any $\mathbf{p}^t = (p_1^t, \dots, p_n^t)$ where the price in market i is p_i^t , and the average revenue for the firm (each price in the average being weighted by its corresponding element in the demand vector $\mathbf{q}(\mathbf{p}^{t-1})$) is at most equal to the *price cap* $\bar{p}^0$ (a component of the n -dimensional *price cap vector* $\mathbf{p}^0 = (\bar{p}^0, \bar{p}^0, \dots, \bar{p}^0) \in \Delta^+$). Mathematically we may write:

$$\frac{\mathbf{p}^t \cdot \mathbf{q}(\mathbf{p}^{t-1})}{\sum_{i=1}^n q_i(\mathbf{p}^{t-1})} \leq \bar{p}^0 \quad (8)$$

or more concisely:

$$(\mathbf{p}^t - \mathbf{p}^0) \cdot \mathbf{q}(\mathbf{p}^{t-1}) \leq 0 \quad (9)$$

which in turn implies that $\mathbf{p}^t$ must lie on or beneath the plane that both contains $\mathbf{p}^0$ and is perpendicular to the demand vector $\mathbf{q}(\mathbf{p}^{t-1})$, and hence by Roy's identity⁵ is parallel to the iso-surplus through $\mathbf{p}^{t-1}$. This requirement for period t prices establishes the recursive aspect of the ARL process. In period t , the myopic firm is expected to search for a $\mathbf{p}^t$ that solves the following problem P:

$$(P) \text{ Maximize } \pi(\mathbf{p}^t) \text{ subject to } (\mathbf{p}^t - \mathbf{p}^0) \cdot \mathbf{q}(\mathbf{p}^{t-1}) = -(\mathbf{p}^t - \mathbf{p}^0) \cdot \nabla V(\mathbf{p}^{t-1}) \leq 0.$$

$\mathbf{p}^t$

Accordingly, it will solve the Lagrangian:

$$L(\mathbf{p}^t, \lambda^t) = \pi(\mathbf{p}^t) - \lambda^t [(\mathbf{p}^t - \mathbf{p}^0) \cdot \mathbf{q}(\mathbf{p}^{t-1})] \quad (10)$$

with Kuhn-Tucker necessary conditions

$$\nabla \pi(\mathbf{p}^t) = \lambda^t \mathbf{q}(\mathbf{p}^{t-1}) \quad (11)$$

$$\lambda^t [(\mathbf{p}^t - \mathbf{p}^0) \cdot \mathbf{q}(\mathbf{p}^{t-1})] = 0 \quad (12)$$

$$\lambda' \geq 0. \quad (13)$$

The period t profit maximizing price vector $\mathbf{p}'$ when the constraint is binding is represented by the unique point of tangency between the constraint plane and the highest iso-profit attainable because $\pi(\mathbf{p})$ is strictly concave. The situation leading to a period t constraint for the single-product / two-market firm is illustrated in Figure 2.

Figure 2. Average-Revenue-Lagged regulation. The constraint rotates about $\mathbf{p}^0$ so that in period t it is parallel to the iso-surplus through $\mathbf{p}^{t-1}$.

IV. Convergence of the Myopic Firm to the Global Profit Maximum

We will show that if the level of the price cap is above the average revenue for profit maximum prices, it is possible that consistently myopic behavior will eventually lead the regulated firm to the profit maximum prices. We should note that it is indeed realistic that such a price cap might be imposed on a firm that previously operated (perhaps under rate-of-return) at zero profit. To illustrate, consider the single-product / two-market firm with the following demand and cost functions:

MARKET 1

Demand: $q_1 = -5.0p_1 + 20$ Variable Cost: $c_1 = 1.0q_1$

MARKET 2

Demand: $q_2 = -6.0p_2 + 40$ Variable cost: $c_2 = 3.0q_2$

Equation (5) gives profit maximum prices $\mathbf{p}^* = (p_1^*, p_2^*) = (2.500, 4.833)$ with corresponding average revenue of 3.887387. Further, if fixed cost is 16.137 then it can be shown that a price cap vector $\mathbf{p}^0 = (3.887, 3.887)$ produces initial profit $0.288 > 0$. Hence, if previous to ARL regulation the firm were constrained to operate at some zero-profit prices that happened to be in the interior of the iso-surplus ellipse passing through this $\mathbf{p}^0$, then a Pareto superior move for the firm and consumers would be to initiate ARL regulation at this $\mathbf{p}^0$. It is expected that such a price movement would be acceptable to all parties involved. Nevertheless, as the following propositions together will support, simple profit maximization within each and every period following such an agreement (which behavior is all that can be reasonably expected of the firm) can ultimately lead to profit maximum prices.

A. PROPOSITION 1

Suppose a two-market firm with linear demands and constant marginal costs operates under ARL regulation with a price cap vector $\mathbf{p}^0$. If the firm is myopic over periods $t = 1, 2, \dots$, then the sequence of price vectors $\{\mathbf{p}^t\}$ assumed by the firm lies along an ellipse in the p_1, p_2 -plane that passes through $\mathbf{p}^0, \mathbf{p}^*$, the efficient price vector $\mathbf{p}^{0, \text{Efficient}}$ whose average revenue is $\mathbf{p}^0$, and the vector $\left(\frac{a_1 c_1 - b_1 + a_2 c_2 - b_2}{2(a_1 + a_2)}, \frac{a_1 c_1 - b_1 + a_2 c_2 - b_2}{2(a_1 + a_2)} \right)$.

See Figure 3.

Proof of Proposition 1:

We assume that the myopic firm moves to a price vector $\mathbf{p}^t$ in period t that is the point of tangency between the period t constraint line and the highest iso-profit attainable (i.e. the constraint line is binding).

Equating the slope of the iso-profit through $\mathbf{p}^t$ from (6) with the slope of the line segment through $\mathbf{p}^0$ and $\mathbf{p}^t$ (i.e. the slope of the period t constraint) yields:

$$\frac{dp_2}{dp_1} = - \frac{2a_1 p_1^t + b_1 - a_1 c_1}{2a_2 p_2^t + b_2 - a_2 c_2} = \frac{\bar{p}^0 - p_2^t}{p^0 - p_1^t} \quad (14)$$

Expanding the second equality about p_1^t and p_2^t gives:

$$\begin{aligned} & 2a_2(p_2^t)^2 + (b_2 - a_2 c_2 - 2a_2 \bar{p}^0)p_2^t - (b_2 - a_2 c_2)\bar{p}^0 \\ &= -2a_1(p_1^t)^2 - (b_1 - a_1 c_1 - 2a_1 \bar{p}^0)p_1^t + (b_1 - a_1 c_1)\bar{p}^0 \end{aligned} \quad (15)$$

which is an ellipse in the p_1, p_2 -plane. Solving (15) with the equation $p_1 = p_2$ implies that the ellipse and Δ^+ intersect at $\mathbf{p}^0$ and

$$\left(\frac{a_1 c_1 - b_1 + a_2 c_2 - b_2}{2(a_1 + a_2)}, \frac{a_1 c_1 - b_1 + a_2 c_2 - b_2}{2(a_1 + a_2)} \right).$$

Also, solving (15) with the equation defining set E from (7) implies that the ellipse and E

intersect at $\mathbf{p}^* = \left(-\frac{(b_1 - a_1 c_1)}{2a_1}, -\frac{(b_2 - a_2 c_2)}{2a_2} \right)$, and the price vector

$$(p_1, p_2) = \left(\frac{a_2 \bar{p}^0 (a_1 c_1 + b_1)(a_1 c_1 + b_1 + a_2 c_2 + b_2) + (a_1 c_1 b_2 - b_1 a_2 c_2)(a_2 c_2 + b_2)}{(a_1)^2 (c_1)^2 a_2 + a_1 (2b_1 c_1 a_2 + (a_2 c_2 + b_2)^2) + (b_1)^2 a_2}, \frac{a_1 \bar{p}^0 (a_1 c_1 + b_1 + a_2 c_2 + b_2)(a_2 c_2 + b_2) - (a_1 c_1 + b_1)(a_1 c_1 b_2 - b_1 a_2 c_2)}{(a_1)^2 (c_1)^2 a_2 + a_1 (2b_1 c_1 a_2 + (a_2 c_2 + b_2)^2) + (b_1)^2 a_2} \right).$$

Substitution of the coordinates of this last ordered pair into the average

revenue function $AR(\mathbf{p}) = \frac{\mathbf{p} \cdot \mathbf{q}(\mathbf{p})}{\sum_{i=1}^2 q_i(p_i)}$ produces $\bar{p}^0$. We therefore label

this ordered pair $\mathbf{p}^{0, \text{Efficient}}$.

Figure 3. Elliptical path of the consistently myopic firm for price cap $\mathbf{p}^0$

B. PROPOSITION 2

Suppose a two-market firm with linear demands and constant marginal costs operates under ARL regulation with a price cap vector $\mathbf{p}^0$. The sequence of profit maximizing price vectors $\{\mathbf{p}^{t, Myopic}\}$ for periods $t = 1, 2, \dots$, not only exists along the elliptical path of Proposition 1, but for a price cap set at $\mathbf{p}^0$ will converge over time to the efficient price vector $\mathbf{p}^{0, Efficient}$ whose average revenue is $\bar{\mathbf{p}}^0$. Moreover, $\mathbf{p}^{0, Efficient}$ is a steady state.

The intuition behind much of the proof of this proposition will follow upon noting that for a situation as that depicted in Figure 3, the iso-surplus curve passing through the period t profit-maximizing price vector $\mathbf{p}^{t, Myopic}$ must be steeper than the iso-profit curve passing through that vector.

Proof of Proposition 2:

Without loss of generality assume $\mathbf{p}^*$ is above the line Δ^+ . Also assume $\mathbf{p}^*$ is not attainable in period t under ARL regulation. Assume linear demands and constant marginal costs.

Let $\mathbf{p}^{t, Myopic}$ be the price vector on the period t constraint line at which

period t profit is a maximum. Then at $\mathbf{p}^{t, Myopic}$ the slope of the iso-profit must be greater than the slope of the iso-surplus curve. This follows because at the period t efficient price vector $\mathbf{p}^{t, Efficient}$ we have

$$-\left(\frac{\partial \pi}{\partial p_1}\right) \bigg/ \left(\frac{\partial \pi}{\partial p_2}\right) \bigg|_{\mathbf{p}^{t, Efficient}} = -\left(\frac{\partial V}{\partial p_1}\right) \bigg/ \left(\frac{\partial V}{\partial p_2}\right) \bigg|_{\mathbf{p}^{t, Efficient}} \quad (\text{i.e. the slope of the}$$

iso-profit and iso-surplus curves are equal) and to traverse the period t constraint from $\mathbf{p}^{t, Efficient}$ to $\mathbf{p}^{t, Myopic}$ we must increase p_1 and decrease p_2 .

This implies decreasing $\frac{\partial \pi}{\partial p_1}$ with increasing $\frac{\partial \pi}{\partial p_2}$ because π is C^2 and

strictly concave. Also, we have increasing $\frac{\partial V}{\partial p_1}$ with decreasing $\frac{\partial V}{\partial p_2}$

because V is C^2 and strictly convex. This leads to the inequality

$$-\left(\frac{\partial \pi}{\partial p_1}\right) \bigg/ \left(\frac{\partial \pi}{\partial p_2}\right) \bigg|_{\mathbf{p}^{t, Myopic}} > -\left(\frac{\partial V}{\partial p_1}\right) \bigg/ \left(\frac{\partial V}{\partial p_2}\right) \bigg|_{\mathbf{p}^{t, Myopic}}. \quad \text{Now the slope of}$$

the period t constraint is $m_t = -\left(\frac{\partial \pi}{\partial p_1}\right) \bigg/ \left(\frac{\partial \pi}{\partial p_2}\right) \bigg|_{\mathbf{p}^{t, Myopic}}$ because the iso-

profit through $\mathbf{p}^{t, Myopic}$ is tangent to the period t constraint. Also the slope

of the period $t + 1$ constraint is $m_{t+1} = -\left(\frac{\partial V}{\partial p_1}\right) \bigg/ \left(\frac{\partial V}{\partial p_2}\right) \bigg|_{\mathbf{p}^{t, Myopic}}$ by

definition of the ARL process (provided $\mathbf{p}^{t, Myopic}$ is assumed by the firm in period t). Hence $m_{t+1} < m_t$ and there is clockwise rotation about $\mathbf{p}^0$ from the period t constraint to the period $t + 1$ constraint. The sequence of price vectors $\{\mathbf{p}^{t, Myopic}\}$ eventually moves rightward and upward from $\mathbf{p}^{1, Myopic}$ along the elliptical path of Proposition 1. Moreover, for any period $t + 1$ we must have

$$\frac{p_2^{0, Efficient} - \bar{p}^0}{p_1^{0, Efficient} - \bar{p}^0} < m_{t+1} \text{ since } m_{t+1} = -\left(\frac{\partial V}{\partial p_1}\right) \bigg/ \left(\frac{\partial V}{\partial p_2}\right) \bigg|_{\mathbf{p}^{t, Myopic}} > \\ -\left(\frac{\partial V}{\partial p_1}\right) \bigg/ \left(\frac{\partial V}{\partial p_2}\right) \bigg|_{\mathbf{p}^{0, Efficient}} = -\left(\frac{\partial V}{\partial p_1}\right) \bigg/ \left(\frac{\partial V}{\partial p_2}\right) \bigg|_{\mathbf{p}^{0, Efficient}} = \frac{p_2^{0, Efficient} - \bar{p}^0}{p_1^{0, Efficient} - \bar{p}^0}.$$

(The second to the last equality is due to the homotheticity property of the iso-surplus family). Hence $\{m_i\}$ is a monotone decreasing sequence

bounded from below. Finally, it is obvious that $\frac{p_2^{0, \text{Efficient}} - \bar{p}^0}{p_1^{0, \text{Efficient}} - \bar{p}^0}$ is the

greatest lower bound for $\{m_i\}$ since for any other vector on the shorter arc from $\mathbf{p}^0$ to $\mathbf{p}^{0, \text{Efficient}}$ of the ellipse of Proposition 1 we must have

$$-\left(\frac{\partial V}{\partial p_1}\right) \Big/ \left(\frac{\partial V}{\partial p_2}\right) < -\left(\frac{\partial \pi}{\partial p_1}\right) \Big/ \left(\frac{\partial \pi}{\partial p_2}\right). \quad \text{Therefore, the sequence } \{m_i\}$$

converges to

$$-\left(\frac{\partial V}{\partial p_1}\right) \Big/ \left(\frac{\partial V}{\partial p_2}\right) \Big|_{\mathbf{p}^{0, \text{Efficient}}} = \frac{p_2^{0, \text{Efficient}} - \bar{p}^0}{p_1^{0, \text{Efficient}} - \bar{p}^0}, \text{ and } \mathbf{p}^{0, \text{Efficient}} \text{ will be a steady}$$

state.

We see then that if a two-market firm with linear demands and constant marginal costs operates under ARL regulation with a price cap satisfying $\bar{p}^0 = \bar{p}^*$ consistently myopic behavior must lead to the global profit maximum $\mathbf{p}^*$, for in this case $\mathbf{p}^{0, \text{Efficient}} = \mathbf{p}^*$ (Figure 4).

Figure 4. A consistently myopic firm moves along an elliptical path toward the global profit maximum when the price cap satisfies $\bar{p}^0 = \bar{p}^*$

Alternatively, if the price cap is set below $\bar{p}^*$, then consistently myopic behavior will lead to the $\mathbf{p}^{0, \text{Efficient}}$ of Figure 3. As the social surplus maximizing vector (c_1, c_2) is situated on E below and to the left of $\mathbf{p}^*$, it is conceivable that a price cap could be selected that induces the myopic firm to marginal cost prices, though firm profits in each period would be negative. Presumably a subsidy to the firm would be provided. In all cases, however, $\mathbf{p}^{0, \text{Efficient}}$ is a steady state.

To illustrate by way of example how myopic behavior under ARL can produce $\mathbf{p}^*$, consider the single-product / two-market firm described previously. Repeated application of (10) – (12) produces the sequence of prices in Table 1 when the price cap satisfies $\bar{p}^0 = \bar{p}^*$.

TABLE 1—A Consistently Myopic Firm Converging to
Global Profit Maximum Prices

Consistently Myopic Behavior (demands $q_1 = -5.0p_1 + 20$ and $q_2 = -6.0p_2 + 40$, cost function $C = q_1 + 3.0q_2 + 16.137$, and price cap $\bar{p}^0 = 3.887387$)			
Period	Myopic Prices ¹	Consumer Surplus	Firm Profit
0	$p_1^0 = 3.887, p_2^0 = 3.887$	$V(\mathbf{p}^0) = 23.205$	$\pi(\mathbf{p}^0) = 0.287$
1	$p_1^1 = 2.464, p_2^1 = 3.935$	$V(\mathbf{p}^1) = 28.280$	$\pi(\mathbf{p}^1) = 10.436$
2	$p_1^2 = 2.368, p_2^2 = 4.599$	$V(\mathbf{p}^2) = 19.475$	$\pi(\mathbf{p}^2) = 14.865$
3	$p_1^3 = 2.483, p_2^3 = 4.811$	$V(\mathbf{p}^3) = 16.083$	$\pi(\mathbf{p}^3) = 15.275$
4	$p_1^4 = 2.500, p_2^4 = 4.833$	$V(\mathbf{p}^4) = 15.713$	$\pi(\mathbf{p}^4) = 15.280$
5	$p_1^5 = 2.500, p_2^5 = 4.833$	$V(\mathbf{p}^5) = 15.708$	$\pi(\mathbf{p}^5) = 15.280$
200	$p_1^{200} = 2.500, p_2^{200} = 4.833$	$V(\mathbf{p}^{200}) = 15.708$	$\pi(\mathbf{p}^{200}) = 15.280$

¹ Prices are rounded to the nearest 0.001.

Subsequent iterations will suggest that a steady state $\mathbf{p}^s$ exists very near our calculated $\mathbf{p}^{200}$. Cowan (1997) asserts, “With ARL regulation consumer surplus is higher in period 1 than with equal prices.” The period 1 increase in consumer surplus from $V(\mathbf{p}^0) = 23.205$ to $V(\mathbf{p}^1) = 28.280$ results from the fact that the iso-surplus passing through $\mathbf{p}^0$, whose slope at $\mathbf{p}^0$ equals the slope of the period 1 constraint, is convex. Additionally, Cowan states, “Steady-state prices when the firm is myopic satisfy the necessary condition for efficiency. Profits are higher, consumer surplus is lower, and welfare is higher than with equal prices.” This proposition

is borne out by the results in Table 1. The steady-state price vector $\mathbf{p}^s$ belongs to the set E of efficient prices. Also, $\pi(\mathbf{p}^s) = 15.280 > \pi(\mathbf{p}^0) = 0.287$, $V(\mathbf{p}^s) = 15.708 < V(\mathbf{p}^0) = 23.205$, and taking social welfare W to be the sum of consumer surplus and firm profit gives $W(\mathbf{p}^s) = 30.988 > W(\mathbf{p}^0) = 23.492$.

For comparison, suppose for the same demand and cost functions the price cap is set at the average revenue corresponding to marginal cost pricing. In this case we have $\bar{p}^0 = 2.189189$. Consistently myopic behavior produces the sequence of prices along the ellipse of Proposition 1 that indeed leads to the social welfare maximum (prices equal to marginal costs) as a steady state. See Table 2. Again we observe Cowan's statements regarding the movement of consumer surplus, profit and welfare.

TABLE 2—A Consistently Myopic Firm Converging to Social Welfare Maximum Prices

Consistently Myopic Behavior (demands $q_1 = -5.0p_1 + 20$ and $q_2 = -6.0p_2 + 40$, cost function $C = q_1 + 3.0q_2 + 16.137$, and price cap $\bar{p}^0 = 2.189189$)			
Period	Myopic Prices ¹	Consumer Surplus	Firm Profit
0	$p_1^0 = 2.189, p_2^0 = 2.189$	$V(\mathbf{p}^0) = 68.341$	$\pi(\mathbf{p}^0) = -27.152$
1	$p_1^1 = 1.522, p_2^1 = 2.414$	$V(\mathbf{p}^1) = 69.607$	$\pi(\mathbf{p}^1) = -24.620$
2	$p_1^2 = 1.230, p_2^2 = 2.655$	$V(\mathbf{p}^2) = 67.461$	$\pi(\mathbf{p}^2) = -21.255$
3	$p_1^3 = 1.105, p_2^3 = 2.813$	$V(\mathbf{p}^3) = 65.506$	$\pi(\mathbf{p}^3) = -18.943$
4	$p_1^4 = 1.050, p_2^4 = 2.900$	$V(\mathbf{p}^4) = 64.268$	$\pi(\mathbf{p}^4) = -17.606$
5	$p_1^5 = 1.024, p_2^5 = 2.950$	$V(\mathbf{p}^5) = 63.576$	$\pi(\mathbf{p}^5) = -16.889$
20	$p_1^{20} = 1.000, p_2^{20} = 3.000$	$V(\mathbf{p}^{20}) = 62.833$	$\pi(\mathbf{p}^{20}) = -16.137$
200	$p_1^{200} = 1.000, p_2^{200} = 3.000$	$V(\mathbf{p}^{200}) = 62.833$	$\pi(\mathbf{p}^{200}) = -16.137$

¹ Prices are rounded to the nearest 0.001.

In contrast to the decreasing consumer welfare movement under ARL noted here, another popular price cap scheme (Laspeyres) is monotone increasing in consumer surplus over time. However, Cowan (1997) also demonstrates that the firm prefers ARL to Laspeyres. Consequently, since social welfare nevertheless increases with ARL, there are good reasons to expect that the firm would be successful in lobbying for the ARL approach.

V. Conclusion

Our use of linear demands and constant marginal costs is an abstraction whereby we have sharpened the already proven result that myopic behavior under ARL leads to efficient prices. We have here demonstrated that under appropriate conditions on demand and cost parameters, the steady-state efficient price vector is well-defined, and under additional conditions on the price cap the steady state will be the global profit maximum. Multiple steady states for a price cap is a feasibility when demand functions are nonlinear, but considerations of such are at best peripheral to the purpose of this paper. Our result has wider application if we make the reasonable assumption that continuous demand functions in general are approximately linear for small price changes. Additionally, though the requirement that the price cap be set at the average revenue of the profit maximum before myopic behavior leads to profit maximum prices may seem at first glance to be a somewhat artificial requirement, we should remember that the only real condition in practice is that the price cap be set at some level equal to or *above* average revenue. If necessary, the firm can always pull inward on its prices. A too-high placement of a price cap is indeed a real-world possibility when the regulatory authority is at a disadvantage in knowing precisely the marginal costs. This is the typical information asymmetry problem between the regulator and the firm. For example, X-inefficiency such as that existing at British Telecom before its privatization (see Bradley and Price 1988), can in fact be the result of firm maneuvering in anticipation of a price cap placement. In all cases of inflated cost data, the regulatory authority is likely to overestimate profit maximum prices.

The possibility that the regulated firm can attain global profit maximum prices when it is simply behaving rationally during each and every regulatory period points to a significant weakness in the ARL approach. In such cases, regulatory intent is undermined without need for strategic behavior.

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Endnotes

1. Cowan (1997) demonstrates that when the firm values future profits, steady state prices under ARL are inefficient and can produce welfare below that prevailing under no regulation. He explains, however, that if the firm does not value profits after the review date for the price cap, then the firm has no incentive in the regulatory period immediately prior to the review to manipulate prices. Moreover, Cox and Isaac (1987) argue that calculating long-term effects of current price selections is very difficult. For these reasons, myopic pricing is a reasonable behavioral assumption for the firm.
2. Sappington and Sibley's analysis involves a consideration of the Federal Communications Commission's price cap regulation of AT&T, an early application of ARL. They demonstrate how the lowering of current usage prices can permit larger entry fees for consumers in later periods.
3. This is a reasonable assumption for many regulated firms where substitution of products or services between markets is difficult or impermissible. Categorization of telephone and natural gas consumers into "commercial" vs. "residential" markets is an example.
4. The dot product used in the definition of $\pi(\mathbf{p})$ is the standard Euclidean inner product of two vectors.
5. Roy's identity in the notation of this paper is $\nabla V(\mathbf{p}) = -\mathbf{q}(\mathbf{p})$.
6. Obtained by use of the computer algebra system *DERIVE*.

