Purchasing Power Parity and Structural Breaks: Rolling, Recursive and Sequential Regressions

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ABSTRACT. Innovative rolling, recursive and sequential regression procedures are employed to examine the evidence concerning purchasing power parity. Two data sets are examined: monthly U.S.-based real exchange rates from 1957-1998 and annual U.K.-based real exchange rates from 1900-1998. Empirical evidence is found for purchasing power parity when structural breaks are included in the annual bilateral exchange rates. Evidence of purchasing power parity and structural breaks is weaker using the monthly data (F3).

I. Introduction

Purchasing power parity is one of the theoretical cornerstones of international finance. Furthermore, many balance of payments and exchange rate determination models are built on the assumption that purchasing power parity (PPP) holds. New innovations in time series analysis have renewed interest in testing the theory of purchasing power parity. However, much of the evidence using these new innovations, including unit root tests and cointegration techniques, is inconclusive due to lack of power or the data length. To increase the power by using a larger data set, the Bretton Woods period must be bridged. Regime changes such as the switch from the fixed exchange rate, Bretton Woods era, to the current floating rate shift begs the question of a structural break in the data mean. This study explicitly tests for a structural break in purchasing power parity using the G-6 (Canada, France, Germany, Italy, Japan and the United Kingdom) bilateral real exchange rates with the United States' dollar.

While low-frequency, annual tests have uncovered evidence on the whole favorable to PPP,[1] high-frequency monthly or quarterly tests have yielded inconclusive results.[2] This paper uses both types of data: high-

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frequency, monthly data from 1957 to 1998 and, for comparison purposes, low-frequency, annual data for the period 1900-1994.

The objective is to test PPP using a procedure suggested by Banerjee et. al. (1992) to detect endogenously any structural breaks that may exist in the real exchange rate series. There is a well-grounded theoretical reasoning for expecting real exchange rates to display structural breaks. For example, Neary (1988) demonstrates that exogenous shocks in technology, tastes or commercial policy are likely to produce permanent shifts in real exchange rates. Moreover, the data sets include observations from two or more exchange-rate regimes, thus necessitating a test for the presence of structural breaks.

Several authors have tested PPP in the presence of structural breaks. Perron and Vogelsang (1992) find evidence in favor of PPP with low-frequency data for the Finish marka/U.S. dollar exchange rate. Using the Zivot and Andrews (1992) procedure, Culver and Papell (1995) test sixteen bilateral real exchange rates with annual data from the gold standard era. In ten cases, they find that a unit root null hypothesis can be rejected at the 5% significance level. In the other six cases, they conclude that four of the real exchange rates can be modeled as stationary around a break in the trend. The purpose of this paper is to use the Banerjee, et. al. (1992) methodology to compare the results from annual low-frequency data with monthly high-frequency data.

The paper is organized into five sections. The second section describes the models and test procedures. Section three presents the results of the monthly data and section four presents the annual data. The final section summarizes the results of the paper.

II. Models and Test Procedures

Purchasing power parity (PPP) is the hypothesis that the domestic price level should equal the exchange rate adjusted foreign price level. According to PPP, bilateral exchange rates adjust to equalize differences in the two countries' price levels. In empirical work, the Consumer Price Index (CPI) is used to proxy the price levels. Deviations from PPP can be expected because of the use of proxies. For relative PPP to hold, it is only necessary that, in the long run, the exchange rate appreciate when the foreign country's price level increases (relative to the domestic price level) and depreciates when the domestic price level increases (relative to the foreign price level).[3]

This study tests PPP via unit root tests of real exchange rates. If PPP were to hold, real exchange rates should not have any permanent deviations but tend to return to an equilibrium level. Therefore, the best prediction of next period's real exchange rate would be this period's real exchange rate.

The real exchange rate, r_t, is defined as:

$$r_t = e_t + p_t - p_t^* \tag{1}$$

where e_t is the natural logarithm of the nominal exchange rate in foreign currency (denominated in domestic currency), p_t is the natural logarithm of the domestic price level and p_t^* is the natural logarithm of the foreign price level in period t. The real exchange rate is considered stationary if it returns to its long-run equilibrium value after a shock. Therefore, for PPP to hold, an autoregression of the real exchange rate must provide evidence that the autoregressive "root" or coefficient is significantly less than one in absolute value. If there is no significant evidence to reject a unit root, the real exchange rate is nonstationary and PPP will not hold.

In order to test this proposition, it is assumed that r_t follows a first-order autoregressive process:

$$r_t = \mu_0 + \mu_1 t + \alpha r_{t-1} + \varepsilon_t \tag{2}$$

where μ_0 and α are constants, and ε_t is normally, identically and independently distributed.

Taking the unconditional expectation of (2) and assuming that $|\alpha| < 1$, the long-run equilibrium real exchange rate is $r_{LR} = \frac{\mu_0}{(1-\alpha)}$. Long-run PPP is violated if either $|\alpha| \geq 1$ or μ_0 and α are not time invariant constants. If $\alpha < 1$, shocks to the system are corrected and the real exchange rate reverts to its mean at the rate $(1 - \alpha)$ per period. Lagged first differences and time trends are often included in (2) to reflect the dynamics of real exchange rates. Otherwise, specification (2) allows only three possibilities: an explosive process, a random walk, or a monotonic adjustment to a constant value.

The rolling, recursive and sequential regression procedures of Banerjee, et. al. (1992) are used to test for a unit root in (2). These tests have been used by Li (1995) with the Nelson-Plosser data set. Rolling and recursive regressions use the transformation of Sims, Stock and Watson (1990) because the elements of the coefficient matrix do not converge at the same rate. Therefore, when there exists one or more unit roots in a vector autoregression (VAR), the coefficients on any intercepts or time trend and their associated t-statistics have nonstandard limiting distributions.[4] The rolling and recursive regressions search for a break in the data by searching for a unit root in subsets of the data set. If the regressions have a unit root in some subsets but then "break" this form and have a stationary root in other subsets, then a break has occurred.

A. THE ROLLING REGRESSIONS MODEL

The rolling regressions procedure tests for a unit root in a fixed subset (δT)of observations (δ<1) where T is the number of observations. The sample is trimmed by $\delta_0 T(\delta_0 = 0.15)$ observations at each end. The first subset begins with the first observation of the data set and ends with the fixed number of observations (δT). Each successive subset begins one time period later and performs the unit root test with the fixed number of observations. The rolling regressions procedure begins at observation δ_0T and uses the next δT observations to test for a unit root (where $\delta = 0.25$). The first observation (δ_0T) is then dropped, the next observation is added ($\delta T + \delta_0 T$) and the test is repeated. The procedure is repeated until the last regression, which includes δT observations up to observation $(1 - \delta_0)$T. The estimated coefficient α is tested for a unit root using the Sims, Stock and Watson (1990) transformation. With a suitable number of lagged first difference terms, the notation in Banerjee, Lumsdaine and Stock (1992) is followed to transform equation (2)[5] as follows:

$$r_t = \boldsymbol{\theta}'\mathbf{Z}_{t-1} + \varepsilon_t \qquad t = 1, ..., T \tag{3}$$

where $\mathbf{Z}_{t-1} = \left[\mathbf{Z}^{1'}_{t-1} Z^2_{t-1} Z^3_{t-1} Z^4_{t-1}\right]'$, and

$$\mathbf{Z}^1_{t-1} = \left(\Delta r_{t-1} - \overline{\mu_0}, \ldots, \Delta r_{t-p} - \overline{\mu_0}\right)',$$

$$Z^2_{t-1} = 1,$$

$$Z^3_{t-1} = \left(r_{t-1} - \overline{\mu_0}(t-1)\right),$$

$$Z^4_{t-1} = t,$$

$$\overline{\mu_0} = E\Delta r_t = \mu_0(1 - \beta(1)), \text{ and}$$

$$\beta(1) = \sum_{i=1}^{p} \beta_i, \beta = (\beta_1, \ldots, \beta_p)$$

The terms in $Z_{t\text{-}1}$ include the p lagged first differences of the real exchange rate $Z^1_{t-1}{}'$, a constant Z^2_{t-1}, the centered real exchange rate Z^3_{t-1}, and the time trend Z^4_{t-1}. The lag length p is chosen so that the residual is white noise. In addition $\theta = (\theta_1 \theta_2 \theta_3 \theta_4)'$ and $\beta_1 = \beta, \theta_2 = \mu_0 + (\beta(1) - \alpha)\overline{\mu_0}, \theta_3 = \alpha, \theta_4 = \mu_1 + \alpha\overline{\mu_0}$. Finally, the residual ε_t is assumed to be a martingale difference sequence.[6]

The rolling OLS estimator of the coefficient vector is:

$$\hat{\theta}(\delta) = (\hat{\theta}_1 \hat{\theta}_2 \hat{\theta}_3 \hat{\theta}_4)' = (\mathbf{L}'\mathbf{L})^{-1}\mathbf{L}'\mathbf{r} \tag{4}$$

where $\mathbf{r} = (r_1, ..., r_{T\delta})'$ and $\mathbf{L} = [\mathbf{Z}_1, ..., \mathbf{Z}_{T\delta}]'$ is a (T x K) matrix, $T\delta$ is the number of observations, K is the number of regressors (p +3), and $\mathbf{Z}$, ..., $\mathbf{Z}_{T\delta}$ are matrices defined in (3). The model is estimated after being trimmed by a fraction δ_0 at either end, where $0 \leq \delta_0 \leq \delta \leq 1$ and δ_0 is chosen to allow a tradeoff between finding breaks in the early and later part of the data set and using enough observations in the regressions to support the Gaussian approximation (Banerjee, Lumsdaine and Stock, 1992, p. 277). Results from the usual distribution theory do not apply when the break point is treated as unknown.[7] Furthermore, the Student t-test converges at the rate $T^{1/2}$, but the Sims, Stock and Watson (1990) transformation "pseudo" Student t-tests converge at the rate of $T^{3/2}$ for the time trend (μ_1) and T for the Dickey-Fuller *tau*-test[8] for a unit root (α). The Dickey-Fuller tau test (τ_{DF}) is therefore calculated as

$$\hat{\tau}_{DF}(\delta) = T(\hat{\theta}_3(\delta) - 1) / \left[\hat{V}_T(\delta)_{33} s^2(\delta)\right]^{\frac{1}{2}} \qquad \delta_0 \leq \delta \leq 1 - \delta_0, \tag{5}$$

where $\hat{\theta}_3(\delta)$ is the estimated coefficient of the "centered" first-lag of the dependent variable $(Z^1_{t-1}), \delta$, is the fraction of the data after trimming, δ_0 is the amount of trimming required, p is the number of lagged error-correction terms, $\hat{V}_T(\delta)_{33}$ denotes the estimate of the diagonal element of the $V_T(\delta) = (Y_T)^{-1} L'L(Y_T)^{-1}$ matrix corresponding to $\hat{\theta}_3$, and Y_T is a diagonal scaling matrix or $Y_T = \mathrm{diag}(T^{1/2}I_p, T^{1/2}, T, T^{3/2})$, I_p is a p-dimensional identity matrix and

$s^2(\delta) = \sum_{t=1}^{[T\delta]} \left[r_t - \hat{\theta}(\delta)' Z_{t-1}\right]^2 / \left([T\delta] - p - 3\right)$. A diagonal scaling matrix is adopted because the elements of θ converge at different rates. It is necessary when drawing inferences on constants or the nonstationary variables, namely those variables with a unit root and/or a time trend coefficient.

B. THE RECURSIVE REGRESSIONS MODEL

The recursive regressions procedure tests for a unit root with a subset of the data that progressively gets larger because the number of observations is increased by one. The first sample uses the initial $[T\delta_0]$ observations. In the empirical section, $\delta_0 = 0.25$. So for the first sample, $\delta = \delta_0 = 0.25$. The sample size, δT, is then expanded by one observation to $\delta_0 T+1$ and the presence of a unit root is again tested. The process is repeated until all observations have been included. Therefore, rather than a fixed number of observations (as in the rolling regression procedure) the recursive regressions increase constantly the number of observations. The model is the same as those in (1) and (2) and the transformation is identical to (3). The Dickey-Fuller tau test is the same as in equation (5).

C. THE SEQUENTIAL REGRESSION MODEL

The sequential regression model also uses an algorithm to search for a potential break endogenously rather than choosing a break point a priori. The model here includes the complete data set series and a dummy variable or time trend depending on whether a shift in the mean or trend value is being tested:

$$r_t = \mu_0 + \mu_1 \tau_{1t}(k) + \mu_2 t + \alpha r_{t-1} + \beta(L)\Delta r_{t-1} + \varepsilon_t \quad t = 1,\dots,T \tag{6}$$

where $\tau_{1t}(k)$ is a dummy variable defined below, t is a variable for time, and β(L) is a lag polynomial of known order p with the roots of 1 - β(L)L outside the unit circle. As in Perron (1990) and Banerjee, Lumsdaine, and Stock (1992) two cases are considered:

Shift in trend: $\tau_{1t}(k) = (t - k)[1(t > k)]$ (7)

Shift in mean: $\tau_{1t}(k) = 1(t > k)$ (8)

where $I(\cdot)$ is the indicator function and k is the time period chosen for the break. Testing the hypothesis that $\mu_1 = 0$ with (7) is equivalent to a test for a change in the slope of the trend whereas using (8) tests whether there is a jump or break in the mean of the time series.

Following Banerjee, Lumsdaine and Stock (1992), the model is transformed as follows:

$$r_t = \boldsymbol{\theta}'\mathbf{Z}_{t-1} + \varepsilon_t \qquad t = 1, ..., T \tag{9}$$

where $\mathbf{Z}_{t-1} = \left[\mathbf{Z}^1_{t-1} Z^2_{t-1} Z^3_{t-1} Z^4_{t-1} Z^5_{t-1}\right]$, $\mathbf{Z}^1_{t-1} = \left(\Delta r_{t-1} - \overline{\mu_0}, \dots, \Delta r_{t-p} - \overline{\mu_0}\right)'$, $Z^2_{t-1} = 1$, $Z^3_{t-1} = \left(r_{t-1} - \overline{\mu_0}(t-1)\right), Z^4_{t-1} = \tau_{1t}(k), Z^5_{t-1} = t,$ and $\overline{\mu_0} = E\Delta r_t = \mu_0\left(1 - \beta(1)\right)$.

There are p lagged first differences of the real exchange rate $\mathbf{Z}^1_{t-1}$ ', a constant Z^2_{t-1}, the centered real exchange rate Z^3_{t-1}, the structural break component Z^4_{t-1}, and the time trend Z^5_{t-1}. The sequential estimator of the coefficient vector, for $\delta_0 \leq \delta \leq 1 - \delta_0$,[9] is:

$$\hat{\theta}(\delta) = \left(\hat{\theta}_1 \hat{\theta}_2 \hat{\theta}_3 \hat{\theta}_4 \hat{\theta}_5\right)' = (\boldsymbol{L}'\boldsymbol{L})^{-1}\boldsymbol{L}'\mathbf{r} \tag{10}$$

with $\mathbf{r} = (r_1, \dots, r_T)'$, $\theta_1 = \boldsymbol{\beta}'$, $\theta_2 = \mu_0 + (\boldsymbol{\beta}(1) - \alpha)\,\overline{\mu_0}$, $\theta_3 = \alpha$, $\theta_4 = \mu_1$, $\theta_5 = \mu_2 + \alpha\overline{\mu_0}$ and $\mathbf{L} = [\mathbf{Z_1}, ..., \mathbf{Z_T}]'$. The residual, ε_t, is assumed to be a Martingale difference sequence. The model in (9) is estimated over the entire sample of T observations. The break point, k, is chosen successively for $k = k_0, k_0+1, \dots, T - k_0$ where $k_0 = [T\delta_0]$ and δ_0 is chosen so as to balance the requirements detecting a trend sufficiently early and sufficient observations to allow appropriate estimation of the coefficients. In the empirical section, δ_0 is set at 0.15. The lag length, p, is chosen so that the residual is white noise. The coefficient of interest for the unit root test (and PPP) is α.

The time period at which the structural break occurs will be chosen on the basis of the supremum of various test statistics. The test statistics computed are a Dickey-Fuller tau test, a Wald test and a Quandt

Likelihood ratio test. The Dickey-Fuller tau test for a unit root ($\theta_3 = 0$) is calculated as follows:

$$\tau_{DF}^{SEQ}(\hat{\theta}) = T\left(\hat{\theta}_3(\delta) - 1\right) / \left[\hat{V}_T(\delta)_{33} s^2(\delta)\right]^{\frac{1}{2}} \quad \delta_0 \le \delta \le 1 - \delta_0 , \tag{11}$$

where $\hat{\theta}_3(\delta)$ is the coefficient of the first-lag of the dependent variable, and the shift parameter occurs at observation [Tδ], p is the number of error-correction terms, $\hat{V}_T(\delta)_{33}$ denotes the diagonal element of the $V_T(\delta)$ matrix corresponding to the unit root coefficient: $V_T(\delta) = (Y_T)^{-1} L'L(Y_T)^{-1}$, $Y_T = \mathrm{diag}(T^{1/2}I_p, T^{1/2}, T, T^{1/2}, T^{3/2})$ in the case of (7) or $Y_T = \mathrm{diag}(T^{1/2}I_p, T^{1/2}, T, T^{3/2}, T^{3/2})$ for (8), I_p is an identity matrix of dimension p and $s^2(\delta) = \sum_{t=1}^{T}\left[r_t - \hat{\theta}(\delta)' Z_{t-1}\right]^2 / (T - p - 4)$.

A Wald test-statistic for testing the significance of the structural break ($\theta_4 = 0$) is computed as follows:

$$F_T(\delta) = \left[W\hat{\theta}(\delta) - w\right]' \left[W(L'L)^{-1}W'\right]^{-1} \left[W\hat{\theta}(\delta) - w\right] / s^2(\delta) \tag{12}$$

where W is a (1 x K) matrix of restrictions, K is the number of regressors and w is a (1 x 1) matrix to test the null hypothesis that $\hat{\theta}_4 = 0$.

A Quandt likelihood ratio test is calculated for each successive break as follows:

$$Q_{LR}(\delta) = \left[-2\ln\lambda(k)\right], \text{ where } \lambda(k) = \frac{(\sigma_{1,k})^k (\sigma_{k+1,T})^{T-k}}{(\sigma_{1,T})^T} \tag{13}$$

where $(\sigma_{T_1,T_2})^2$ is the Gaussian maximum likelihood estimator of the regression error variance calculated over observations $T_1, ..., T_2$. This test is used to test for a break in any or all of the coefficients. "The LR statistic is calculated for each possible break point, and the Quandt LR statistic, $Q_{LR,}$ is the maximum of these." (Banerjee et. al., 1992, p. 276) The critical values are calculated by Monte Carlo simulation and reported in Banerjee et. al. (1992).

The requirement that enough observations be included to support the Gaussian approximation including early and late observations to capture possible breaks early and late in the sample led Banerjee et. al. to "trim" the Q_{LR} statistic by 25% at each end and the $F_T(\delta)$ and τ_{DF}^{SEQ} by 15% at each end. This paper follows their suggestion.

III. MONTHLY DATA AND RESULTS

The United States is the base country for the monthly empirical results, which cover the period January 1957 through December 1998. The Consumer Price Index (line 63 of the IMF's International Financial Statistics, IFS) is the proxy for the price levels and the nominal exchange rate is in foreign currency units per dollar (line *rf* of the IFS).

Standard Augmented Dickey-Fuller (ADF) tests (results are available on request) fail to reject the hypothesis that the real exchange rate follows a random walk. However, the null hypothesis that first differences of the real exchange rates have a unit root was rejected. Thus, the real exchange rate is stationary in its first differences and integrated of order one.

Therefore the next procedure was to apply the previously described rolling, recursive and sequential methodology. Tables 1 and 2 show the ADF tau-test statistics for the rolling and recursive tests, respectively.

TABLE 1–Rolling regression tau tests for a unit root using monthly data

	One Lag			Zero Lags		
Country	Min Month	τ^{max}CPI	τ^{min}CPI	Min Month	τ^{max}CPI	τ^{min}CPI
Canada	Feb 1960	0.25	-2.39	Nov 1959	0.39	-2.34
France	July 1980	-0.27	-2.53	July 1980	0.36	-2.37
Germany	July 1980	1.09	-2.49	July 1980	2.54	-2.32
Italy	July 1980	-0.60	-2.26	July 1980	0.03	-1.49
Japan	Oct 1978	0.35	-2.49	Oct 1978	0.55	-2.24
United Kingdom	Jan 1981	0.34	-2.19	Jan 1981	0.94	-1.88

Note: Min Month is the break month, the month that tau^{min} is at the value reported. The τ^{min} critical values for 500 observations are -4.55, -4.79 and -5.00 for the 10%, 5% and 2.5% significance levels respectively. The τ^{max} critical values for 500 observations are -1.25, -1.47 and -1.62 for the 10%, 5% and 2.5% significance levels respectively. These are calculated by Monte Carlo simulation by Banerjee, Lumsdaine and Stock (1992, p. 277).

TABLE 2–Recursive Regression tau Tests for a Unit Root Using Monthly Data

	One Lag			Zero Lags		
Country	Min Month	τ^{max}CPI	τ^{min}CPI	Min Month	τ^{max}CPI	τ^{min}CPI
Canada	Dec 1991	-0.82	-1.18	Dec 1991	-0.62	-1.24
France	July 1969	-3.36***	-3.74	July 1969	-4.18**	-4.44***
Germany	Apr 1985	-2.79***	-3.08	Apr 1985	-2.03**	-2.80
Italy	Apr 1985	-1.77*	-1.82	Apr 1985	-1.82*	-1.88
Japan	Dec 1971	-3.49***	-3.89*	Dec 1971	-3.42***	-3.95*
United Kingdom	Mar 1985	-1.39	-1.62	Apr 1985	-4.00***	-5.06***

Note: Min Month is the break month, the month that tau^{min} is at the value reported. The τ^{min} critical values for 500 observations are -3.88, -4.18 and -4.42 for the 10%, 5% and 2.5% significance levels respectively. The τ^{max} critical values for 500 observations are -1.66, -1.92 and -2.17 for the 10%, 5% and 2.5% significance levels respectively. These are calculated by Monte Carlo simulation by Banerjee, Lumsdaine and Stock (1992, p. 277).

The two values reported, the maximum and minimum tau test statistics, are the highest and lowest values of the *tau*-test in (5) over the dates searched. The critical values, which depend on the sample size, but not the number of lags, were computed by Monte Carlo simulation by Banerjee et. al. (1992). With a sample size of 492 observations, the critical values are reported for 500 observations.

The standard Student t-test critical values are 1.96 and 2.326 for 2.5% and 1% one-tailed test significance levels, respectively. Therefore, in Table 1 all of the one-lag test statistics would be significant at the 2.5% and most at the 1% significance level. However, when using the BLS (1992) Monte Carlo simulation critical values, none of the test statistics are found to be significant at even the 10% level. It should be pointed out that the data subset used in the rolling regressions case spans only fifteen years (or 180 observations).

The recursive regression tests found evidence of mean reversion for the franc/U.S. dollar and yen/U.S. dollar exchange rates from the recursive tau-statistics in Table 2. The breaks occur near the end of the Bretton Woods era in July 1969 and December of 1971, respectively. The break occurs in the one-lag ADF statistics, as well as the zero-lag Dickey-Fuller tau-tests.[10] German and Italian maximum tau statistics are also significant, the break occurring in April of 1985, the date of the Plaza Accord. The pound/U.S. dollar exchange rate with zero lags is also highly significant and coincides with the Plaza Accord in 1985[11]. The recursive results reject the random walk representation (find evidence for PPP) in thirteen of the twenty-four test statistics. Interestingly, there exists more evidence of a break during the Plaza Accord than at the end of the Bretton Woods era. Next, the sequential model is used to test for a structural break in the data.

Four statistics of interest to this analysis are calculated for the sequential regression model: the maximum value for the F_T statistic in (12) that tests the hypothesis that the mean shift is zero ($\mu_1 = 0$), or F^{max}; the Dickey-Fuller tau statistic in (11) evaluated at period k that maximizes the F_T statistic, or τ^{max}; the minimal Dickey-Fuller tau statistic over all k, or τ^{min}; and the maximum Quandt Likelihood Ratio statistic, or Q_{LR} in (13). Table 3 reports these four statistics for a shift in the mean as shown in equation (7) (results of the shift in the trend are available on request). These tables also show the date (k) at which the maximum F_T statistic (F^{max}) is observed.

The Quandt likelihood ratio statistic is significant for all countries and all real exchange rates except France. The breaks in all of the Quandt statistics occur during the 1969-1973 end of the Bretton Woods era. Banerjee, Lumsdaine and Stock (1992 p. 279) state that the Quandt likelihood ratio test statistic is a powerful and reliable diagnostic tool for detecting a break over all of the coefficients. While Banerjee, Lumsdaine and Stock do not report critical values for the one lag case, the four-lag 2.5% critical value is 37.00, which is significantly below the other five Quandt test statistics in Table 3. Therefore, there is conflicting evidence between the recursive regressions results (breaks during the Plaza Accord of 1985) and the sequential regression results (breaks occurring during the end of the Bretton Woods era).

Table 3 reports test results with one and zero autoregressive lagged first difference terms included in the model. A significant structural break, the maximum F_T statistic (F^{max}), is detected in the one-lag and zero-lag models on December 1991 in the Canadian-based real exchange rate (at the 10% level). However, ten of the twelve bilateral real exchange rates do not exhibit mean shift structural breaks. None of the Augmented Dickey-Fuller *tau* statistics evaluated at the break (τ^{max}) are significant at the 5% level. Furthermore, none of the trend-shift statistics (results available on request) are significant.

TABLE 3–Sequential Test Statistics for a Mean Shift Using Monthly Data

			One	Lag				Zero	Lags	
Country	k	F^{max}	τ^{max}	τ^{min}	$Q_{LR}(1)$	k	F^{max}	τ^{max}	τ^{min}	Q_{LR} (0)
Canada	12/91	18.92*	-2.58	-2.58	59.4***	12/91	18.87*	-2.59	-2.59	57.9***
France	4/85	7.89	-2.89	-2.89	31.4	4/85	6.67	-2.58	-2.58	33.3***
Germany	4/85	8.17	-2.67	-2.67	301.1***	4/85	6.27	-2.50	-2.50	302.8***
Italy	4/85	11.75	-3.58	-3.60	277.9***	4/85	10.80	-3.44	-3.44	279.4***
Japan	9/71	9.14	-2.94	-2.94	221.2***	9/71	7.20	-2.93	-2.93	232.1***
United States	3/85	12.71	-3.23	-3.23	141.8***	3/85	12.98	-3.07	-3.07	122.9***

Note: k is when the structural break date occurs. The critical values are -4.49, -4.77 and -5.05 for the τ^{max} mean shift statsitics and are -4.51, -4.78 and -5.05 for the τ^{min} mean shift statistics for the 10%, 5% and 1% significance levels respectively. The critical values for the F_T mean shift statistics are 16.78, 18.99 and 21.26 for the 10%, 5% and 1% significance levels respectively. For $Q_{LR}(0)$ they are 25.19 (10%), 27.80 (5%) and 30.42 (2.5%), for $Q_{LR}(4)$ the 2.5% critical value is 37.00. These are calculated by Monte Carlo simulation by Banerjee, Lumsdaine and Stock (1992, p. 278). $Q_{LR}(1)$ statistics are not calculated by the authors, however, our statistics (except France) are substantially above the four lag case. These values are calculated with a sample size of 500 (our sample size is 492).

Thus, in mean and trend shift cases (one and zero lags), neither the τ^{max} nor the τ^{min} statistics are significant. So while two of the representations have significant structural breaks, the tau tests fail to reject the hypothesis that real exchange rates have a unit root and find little evidence for PPP with sequential tests and monthly data.

IV. Annual Data and Results

The data set consists of twentieth century (1990 – 1998) annual real sterling exchange rates for the G-7 minus Germany.[12] The U.K. pound serves the base currency because most other long-term studies of PPP[13] employ the U.S. dollar and as a contrast to observe whether the Plaza Accord (April 1985) breaks exist in the U.K. exchange rates as well. Italy does not have data prior to 1917, thus leaving 84 observations in this data set. All data are from Liesner (1989) and were updated via the IFS.

Standard Augmented Dickey-Fuller test results (available on request) show that the unit root null can be rejected for five of the ten real exchange rates. There is stronger evidence that PPP holds using annual, low-frequency data.

Tables 4-5 show the results of the rolling and recursive regressions for the cases when one lag and zero lags are included. Only one of the ten tau-test statistics rejects the unit root hypothesis when one lag is included in a rolling regression model. When no lags are included only one additional case becomes significant. The unit root hypothesis is rejected for Italy at the 1% level. There is not much additional support for PPP using annual data rather than monthly data in the rolling regressions model.

In the recursive case (Table 5), eight of ten tests are significant at the 10% level when one lag is included. When a structural break is endogenously included, evidence is found, around the World War II era, for Purchasing Power Parity at the 1% significance level for France (1943), Japan (1947) and the United States (1949). Alternatively, evidence for PPP is also found in Italy's (1932) exchange rate when a structural break is included. 1932 was a very vibrant time for the fascist Italian economy and leader which may explain the structural break. By 1940, Italy's economy was already in decline when they joined the Third Reich. There is a noted decrease in the number of significant cases when the lagged first difference is dropped, as only Italy, Japan and the United States continue to reject PPP at the 10% level.

TABLE 4–Rolling Regression tau Tests for a Unit Root, Annual Data

	One Lag			Zero Lags		
Country	Min Month	τ^{max}CPI	τ^{min}CPI	Min Month	τ^{max}CPI	τ^{min}CPI
Canada	1986	0.03	-2.30	1986	-0.48	-2.40
France	1970	-0.25	-2.11	1971	-0.61	-2.45
Italy	1973	0.19	-5.76***	1973	0.62	-8.67***
Japan	1980	0.01	-4.03	1980	-0.19	-3.94
United Kingdom	1946	-0.09	-3.14	1946	-0.14	-3.05

Note: Min Year is the break year, the year that *tau*^{min} is at the value reported. The τ^{min} critcal values for 100 observations are -4.71, -5.01 and -5.29 for the 10%, 5% and 2.5% significance levels respectively. The τ^{max} critical values for 100 observations are -1.31, -1.49 and -1.66 for the 10%, 5% and 2.5% significance levels respectively. These are calculated by Monte Carlo simulation by Banerjee, Lumsdaine and Stock (1992, p. 277).

Though there are differences in these four rolling and recursive regression models (the monthly and annual data), little additional conclusive support for PPP is found when no lags are included. However, when the lagged real exchange rate is included, substantial evidence is found for mean reversion (support for PPP). The investigation concludes using the sequential regressions model to search for evidence of a structural break within the unit root model.

TABLE 5–Recursive Regressions tau Tests for a Unit Root, Annual Data

	One Lag			Zero Lags		
Country	Min Month	τ^{max}CPI	τ^{min}CPI	Min Month	τ^{max}CPI	τ^{min}CPI
Canada	1986	-1.09	-2.26	1984	-1.52	-2.66
France	1943	-2.98***	-5.43***	1943	-1.09	-3.82
Italy	1932	-3.74***	-4.25*	1932	-2.48**	-4.35***
Japan	1947	-2.23*	-4.88***	1947	-2.99***	-3.69
United Kingdom	1949	-2.80***	-5.12***	1949	-1.88*	-3.88

Note: Min Year is the break year, the year that *tau*^{min} is at the value reported. The τ^{min} crtical values for 100 observations are -4.00, -4.33 and -4.62 for the 10%, 5% and 2.5% significance levels respectively. The τ^{max} critical values for 100 observations are -1.73, -1.99 and -2.21 for the 10%, 5% and 2.5% significance levels respectively. These are calculated by Monte Carlo simulation by Banerjee, Lumsdaine and Stock (1992, p. 277).

The annual sequential regression results are in Table 6. The data are trimmed by the same percentages as the monthly data resulting in a smaller data set. Therefore, the Banerjee, et. al. (1992) critical values for a sample size of 100 are used. As with the monthly data, the Quandt likelihood ratio test results in highly significant statistics with the exception of France. The structural breaks are indicated at the end of the Bretton Woods era. Monte Carlo simulation for the single lag case of the Quandt likelihood ratio test is outside of the scope of this paper. However, the highest critical value (2.5%) reported by Banerjee, et. al. (1992) for the four lag case is 37.25. Therefore, confidence is high in assuming that Monte Carlo simulation would confirm a value less than 37.25 for the 2.5% critical value and significances are assigned accordingly.

The Canadian dollar/sterling and U.S. dollar/sterling exchange rates show significant breaks coincident with the postwar era (1949) at the 10% significance level. In addition, five of the 10 real exchange rates examined in Table 6 provide evidence to support PPP, that is they reject the hypothesis of a unit root. The United States in the one lag case and Italy in the zero lagged first differences case reject a unit root at the 1% significance level. The other three, Italy (one lag), France (zero lags) and Canada (one lag) reject a unit root (find evidence in favor of PPP) at the 10% significance level.

The evidence based on low-frequency, annual data is more favorable to PPP than that from high-frequency, monthly data in section three. The recursive and sequential tests find evidence for PPP and while not conclusive, definitely significant. In combination, evidence for Purchasing Power Parity is found in all five bilateral exchange rates.

V. Conclusions

The real exchange rates for the Group of Seven countries are examined to test for mean reversion. Monthly data spanning 1957-1998 and annual data covering 1900-1998 are employed to test the theory of purchasing power parity (PPP).

Three types of models are used in the analysis: rolling regression models, recursive regression models, and sequential regression models. The mixed evidence for PPP, even over low-frequency data, led to the current research. Perhaps a break in the data distorts the evidence and lack of conclusive evidence. The models employed search endogenously for a structural break in the bilateral real exchange rates while testing simultaneously for PPP (a unit root hypothesis).

TABLE 6–Sequential Test Statistics, for a Mean Shift, Annual Data

			One	Lag				Zero	Lags	
Country	k	F^{max}	τ^{max}	τ^{min}	$Q_{LR}(1)$	k	F^{max}	τ^{max}	τ^{min}	$Q_{LR}(0)$
Canada	1949	16.88*	-4.63*	-4.63*	44.59***	1948	15.59	-4.41	-4.41	44.94***
France	1943	8.45	-4.21	-4.21	35.42	1942	9.92	-4.58*	-4.58*	22.87
Italy	1932	9.54	-4.56*	-4.56*	39.33***	1932	7.62	-5.22***	-5.22***	31.50***
Japan	1947	13.87	-2.66	-2.78	41.91***	1947	12.51	-3.17	-3.17	40.14***
United Kingdom	1949	17.72*	-4.83**	-4.83**	44.12***	1949	7.91	-3.92	-3.92	36.85***

Note: k is the date the structural break occurs. The critical values are -4.52, -4.80 and -5.07 for the τ^{max} mean shift statistics and are -4.54, -4.80 and -5.07 for the τ^{min} mean shift statistics for the 10%, 5% and 1% significance levels respectively. The critical values for the F_T mean shift statistics are 16.20, 18.62 and 20.83 for the 10%, 5% and 1% significance levels respectively. For $Q_{LR}(0)$ they are 23.86 (10%), 26.45 (5%) and 28.96 (2.5%). These are calculated by Monte Carlo simulation by Banerjee, Lumsdaine and Stock (1992, p. 278). $Q_{LR}(1)$ statistics are not computed by Banerjee, Lumsdaine and Stock (1992), however as with the monthly data, 37.25 is the 2.5% $Q_{LR}(4)$ critical value and thus the assigned significance values above. These values are calculated with a sample size of 100 (our sample size is 99).

The rolling regression models test for a unit root in a fixed subset of the data. The model fails to reject the unit root hypothesis for all of the countries using the monthly data set. With annual data, the model is able to reject (find evidence for PPP) only for the lira/sterling exchange rate.

The recursive regression model also tests a subset of the data that progressively becomes larger. The unit root null is rejected for the French, German, Italian, Japanese and United Kingdom real exchange rates with monthly data. As for the annual data, in addition to strong evidence of mean reversion for the French, Italian, Japanese and the U.S. (pound sterling) real exchange rates.

The sequential regressions models use the entire data set and include a structural break variable sequentially to determine endogenously whether a break in the data set exists. Though evidence of a break exists for Canada, the model is unable to reject the null hypothesis of a unit root in any of the cases with monthly data. The annual data are more favorable to PPP with 50% of the cases rejecting the unit root null. In addition, structural breaks are found for Canada and the U.S. after World War II in 1949.

The annual bilateral, real exchange rates provide evidence in favor of purchasing power parity in each of the cases. Further research could focus on panel data, especially using monthly data, combining the Group of Seven to test for structural breaks and evidence for purchasing power parity.

References

Abuaf, N. and Jorion, P. "Purchasing power parity in the long run." *The Journal of Finance*, 1990, 45, 157-174.

Ardeni, P. G. and Lubian, D. "Is there a trend reversion in purchasing power parity?" *European Economic Review*, 1991, 35, 1035-1055.

Banerjee, A.; Lumsdaine, R. and Stock, J. "Recursive and sequential tests of the unit-root and trend-break hypotheses: theory and international evidence." *Journal of Business and Economic Statistics*, 1991, 10, 271-287.

Cheung, Y. and Lai, K. S. "Long-run purchasing power parity during the recent float." *Journal of International Economics*, 1993, 34, 181-192.

Christiano, L. "Searching for a break in GNP." *Journal of Business and Economic Statistics*, 1992, 10, 237-250.

Culver, S. and Papell, D. "Real exchange rates under the gold standard: Can they be explained by the trend break model?" *Journal of International Money and Finance*, 1995, 14, 539-548.

Dickey, D. A. and Fuller, W. A. "Distribution of the estimators for autoregressive time series with a unit root." *Journal of the American Statistical Association*, 1979, 74, 427-431.

Diebold, F. X., Husted, S. and Rush, M. "Real exchange rates under the gold standard." *Journal of Political Economy*, 1991, 99, 1252-1271.

Enders, W. "ARIMA and cointegration tests of PPP under fixed and flexible exchange rate regimes." *Review of Economics and Statistics*, 1988, 70, 504-508.

Evans, G. W. "Output and unemployment dynamics in the United States: 1950-1985." *Journal of Applied Econometrics*, 1989, 4, 213-237.

Frankel, J. A. and Rose, A. K. "A panel project on purchasing power parity: Mean reversion within and between countries." *Journal of International Economics*, 1996, 40, 209-224.

Froot, K. A. and Rogoff, K. "Perspectives on PPP and long-run real exchange rates." NBER Working Paper Series, 1994, Working Paper No. 4952.

Glen, J. D. "Real exchange rates in the short, medium, and long run." *Journal of International Economics*, 1992, 33, 147-166.

Hegwood, N. and Papell. "Quasi Purchasing Power Parity." Unpublished manuscript, 1996.

Jorion, P. and Sweeney, R. "Mean reversion in real exchange rates: Evidence and implications for forecasting." *Journal of International Money & Finance*, 1996, 15, 535-550.

Kim, Y. "Purchasing power parity in the long run: A cointegration approach." *Journal of Money, Credit, and Banking*, 1990, 22, 491-503.

Li, Hongyi. "A reexamination of the Nelson-Plosser data set using recursive and sequential tests." *Empirical Economics*, 1995, 20, 501-518.

Liesner, T. *One hundred years of economic statistics.* New York, NY: Facts on File, 1989.

Mark, N. C. "Real and nominal exchange rates in the long run: An empirical investigation." *Journal of International Economics*, 1990, 28, 115-136.

Neary, P. "Determinants of the equilibrium real exchange rate." *American Economic Review*, 1988, 78, 210-215.

Perron, P. "Testing for a unit root in a time series with a changing mean." *Journal of Business and Economic Statistics,* 1990, 8, 153-162.

Perron, P. and Vogelsang, T. J. "Nonstationarity and level shifts with an application to purchasing power parity." *Journal of Business and Economic Statistics*, 1992, 10, 301-320.

Phylaktis, K. and Kassimatis, Y. "Does the real exchange rate follow a random walk? The Pacific Basin perspective." *Journal of International Money and Finance*, 1994, 13, 476-495.

Sims, C. A., Stock, J. H., and Watson, M. W. "Inference in linear time series models with some unit roots." *Econometrica*, 1990, 58, 113-144.

Zivot, E. and Andrews, D. W. K. "Further evidence on the great crash, the oil-price shock, and the unit-root hypothesis." *Journal of Business and Economic Statistics*, 1992, 10, 251-269.

Endnotes

1. For example Abuaf and Jorion (1990), Diebold, Husted and Rush (1991), Glen (1992), Kim (1990), Perron and Vogelsang (1992).
2. For example Abuaf and Jorion (1990), Ardeni and Lubian (1991), Cheung and Lai (1993), Enders (1988), Mark (1990), and Phylaktis and Kassimatis (1994).
3. Froot and Rogoff (1994, p.5) state that "the bulk of the empirical literature focuses on testing *relative consumption-based PPP* ... which requires that *changes* in relative price levels be offset by changes in the exchange rate."
4. This transformation is necessary only when one considers joint hypotheses, for example the Wald and Quandt tests, of parameters converging at different rates.
5. Δr_t is the i^{th} lagged first difference of the real exchange rate. These terms are added in the Augmented Dickey-Fuller tests.
6. A martingale difference sequence is a sequence of scalars that have a mean of zero over all of the variable's values and an expectation of zero given all of the past and present values of the variable. Thus, $\{Y_t\}_{t=1}^{\infty}$ is a martingale difference sequence if $E(Y_t) = 0$ and $E(Y_t \;\; Y_{t-1}, Y_{t-2}, \ldots, Y_1) = 0$. This condition is stronger than assuming that Y_t is serially uncorrelated. A serially uncorrelated sequence cannot be forecast on the basis of a linear function of its past values. This does not rule out the possibility that higher moments such as $E\left(Y_t^2 \middle| Y_{t-1}, Y_{t-2}, \ldots, Y_1\right)$ might depend upon past values. Thus $\{Y_t\}_{t=1}^{\infty}$ may not be independent so Banerjee, Lumsdaine and Stock (1992) add an additional assumption, which specifies that higher moments must be finite.
7. Banerjee, Lumsdaine and Stock (1992, p. 272) develop a distribution theory for a series of statistics evaluated over a range of possible break dates. "This permits analyzing the distribution of continuous functionals of these statistics--for example, the maximum of the sequence of unit-root test statistics, one for each possible break date. Christiano (1988) and Evans (1989) recognized the nonstandard nature of these distributions and used numerical simulations to examine extrema of sequences of test statistics."
8. See Dickey and Fuller (1979) for a full description of this test.
9. Thus δ is a subset of the data set between δ_0 and $1 - \delta_0$.
10. The coefficients are 0.8926 and 0.8825 for 4 lags and no lags, respectively.
11. The Plaza Accord of 1985, signed by the G-5 was designed to depreciate the U.S. dollar vis-à-vis the Japanese yen and German Deutsche Mark. The concerted effort of these central banks was successful resulting in a 51% depreciation over the next two years.
12. Germany is excluded because of missing data during the war years. Other researchers (e.g. Hegwood and Papell, 1996; Kim, 1990) also exclude Germany.
13. See for example Abuaf and Jorion, 1990; Cheung and Lai, 1994; Kim, 1990.