Tests for Asymmetric Threshold Cointegration with an Application to the Term Structure

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ABSTRACT. A number of recent studies have found evidence of a nonlinear term structure relationship in the U.S. This paper extends earlier work and develops tests of the null hypothesis of no cointegration that allow for linear or asymmetric (or symmetric) threshold cointegration under the alternative hypothesis. Applied to the residuals from regressions of long-term interest rates on short-term interest rates for eleven countries, our tests reject the no cointegration hypothesis for eight of the countries in our sample. We find that for a majority of the countries, the adjustment of the long-term rate to disequilibrium is fastest when the equilibrium error is above the upper threshold, and the adjustment of the short-term rate to disequilibrium is fastest when the equilibrium error is below the lower threshold. (C22; E43)

I. Introduction

In this paper we develop tests of the null hypothesis of no cointegration against the alternative hypothesis of threshold cointegration and apply these methods to the term structure of interest rates. The type of threshold cointegration model used in this paper is linked to the concept of threshold cointegration as introduced by Balke and Fomby (1997), which is an extension of the Engle-Granger approach. In a threshold cointegrating relationship the equilibrium error from a cointegrating regression is globally stationary, however, it behaves as an I(1) process within a central band. This allows for the possibility that costs of adjustment may prevent economic agents from behaving in such a way as to maintain the equilibrium relationship between two I(1) variables when deviations from that equilibrium are small. However, when deviations exceed a critical threshold, actions are taken by economic agents (e.g. through arbitrage when transaction costs are exceeded) and the long-run relationship is restored. Threshold autoregressive (TAR) models for the equilibrium error can be used to capture this type of nonlinear adjustment.

This study extends earlier work in a number of ways. Firstly, none of

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the earlier studies applying the TAR methodology to cointegration, in particular, Enders and Granger (1998), Enders and Siklos (2001), Balke and Fomby (1997) and Hansen and Seo (2002) consider testing the null hypothesis of *no cointegration* against the alternative hypothesis of *threshold cointegration* as defined by Balke and Fomby (1997). While Enders and Siklos (2001), and Hansen and Seo (2002) test the null hypothesis of no cointegration, under the alternative hypothesis they allow for a stationary TAR adjustment around a linear attractor. Their models do not explicitly entertain the possibility of a central regime of I(1) behavior.

Secondly, previous studies assume *symmetric* mean reversion under the alternative hypothesis, in the sense that although they allow for TAR nonlinearity under the alternative hypothesis, an implicit assumption is that the degree of mean reversion of the equilibrium error is the same for positive deviations of the series from its attractor as for negative deviations of the same proportionate amount. The tests developed in this paper allow for both *symmetric* or *asymmetric* nonlinear mean reversion under the alternative hypothesis (or linear mean reversion). Hence, our tests are considerably more general. Another recently proposed test of no cointegration allowing for nonlinear cointegration under the alternative is that of Kapetanios Shin, and Snell (2006). Their tests are similar to those developed in this paper, although the models they employ are smooth transition autoregressive (STAR) rather than the TAR models employed here.

Finally, we apply our testing methodology to the term structure of interest rates. A number of the earlier papers that we extend use the term structure as an empirical application, however, these papers tend to restrict their empirical analysis to data on U.S. interest rates, while we apply the technology developed to data for eleven OECD countries, including the U.S. The presence of TAR nonlinearity in financial time series data is thought to be a consequence of transactions costs affecting the behaviour of arbitrageurs. It is important therefore to discover if the presence of threshold nonlinearity is just a specific feature of U.S. term structure data, or a feature of term structure data for other countries.

The tests proposed in this paper may have useful applications to other empirical relationships where economic theory suggests that there should be a long-run cointegrating relationship, but standard testing procedures have been unable to uncover such a relationship. Examples include the purchasing power parity hypothesis, which often is found not to hold when subject to standard unit root or cointegration tests, as well as the long-run relationship between stock prices and dividends, which present value theory suggests should be cointegrating, but which is typically found not to be when conventional unit root or cointegration tests are used. This may be a consequence of the true data generating process being more general in nature as our methodology allows for in this paper.

The seminal empirical work on the term structure of interest rates employing the cointegration methodology is the work of Campbell and Shiller (1987). These authors show that if short-term and long-term nominal interest rates are both integrated of order one, I(1), these series should be cointegrated with a cointegrating vector of unity, yielding a stationary term spread. Campbell and Shiller employed the then recently developed Engle-Granger methodology in their study and found some evidence of cointegration. Applied to the term structure, the Engle-Granger approach involves testing for a unit root in the short-term and long-term rates and then testing for a unit root in the residuals from a long-run regression of the long-term interest rate on the short-term interest rate (*or vice-versa*). Typically the augmented Dickey-Fuller (ADF) (Dickey and Fuller, 1979) test is used.

As mentioned above, several authors have investigated the possibility of a nonlinear long-run relationship between short-term and long-term interest rates. Enders and Granger (1998) develop tests of the null hypothesis of a unit root that allow under the alternative hypothesis for a TAR process around a linear attractor (see Kapetanios and Shin, 2006, for a more recent unit root test allowing for threshold nonlinearity under the alternative). Enders and Granger (1998) apply these tests to monthly univariate time series data on the term spread for the U.S., defined as the long-term rate minus the short-term rate (thus imposing a cointegrating vector of unity) and find statistically significant evidence against the unit root null. This is an interesting result since conventional unit root tests that assume linear mean reversion under the alternative hypothesis usually do not reject the unit root null hypothesis when applied to U.S. term structure data.

Following Enders and Granger (1998), a number of other studies have used the term structure of interest rates in the U.S. as an empirical example for demonstrating the nonlinear methods they have developed. For example, Enders and Siklos (2001) extend the work of Enders and Granger (1998). They propose that two series may exhibit a cointegrating relationship but the adjustment to equilibrium could follow a stationary

TAR process around a linear attractor. Enders and Siklos (2001) extend the *t*- and *F*- tests developed by Enders and Granger (1998) to the case of testing the null hypothesis of no cointegration between two variables. They simulate appropriate critical values and also apply their tests to the term structure of interest rates for the U.S. Thus, the empirical application in Enders and Siklos (2001) is the same as that in Enders and Granger (1998), but the former estimate the cointegrating vector rather than imposing it to be unity. Another study that has used U.S. term structure data in its empirical application is the work of Hansen and Seo (2002) who derive the asymptotic distribution of a Sup-*LM* test for testing the null of linear cointegration against the alternative of nonlinear cointegration (defined as stationary TAR adjustment around a linear attractor).

Testing the null hypothesis of no cointegration using TAR models is hampered by the presence of unidentified parameters (the thresholds) under the null hypothesis. Consequently, conventional analytical procedures cannot be used to derive the limiting distributions. The problem of unidentified thresholds under the null hypothesis is one also faced by the authors mentioned above. They circumvent this problem by simulating finite sample critical values using Monte Carlo simulation. For example the Monte Carlo procedure of Enders and Siklos (2001) involves generating independent I(1) processes, estimating the equilibrium error from a regression of these independent processes, and then estimating the relevant test statistic. We also use this procedure to build up the empirical distribution of the test statistics proposed. Note that since the thresholds are unidentified under the null, the estimates of the thresholds in this procedure are essentially meaningless, however, Monte Carlo experiments reported by Enders and Granger (1998) show that this procedure leads to a test statistic that has substantially more power than conventional Dickey-Fuller procedures for testing against TAR adjustment.

In the next section of this paper we present a brief review of the empirical literature on the term structure of interest rates. In Section 3 we develop our tests of the null hypothesis of no cointegration which have threshold cointegration as the alternative hypothesis and discuss the simulation of critical values. Section 4 focuses on the empirical results from applying these tests to our data on the term structure of interest rates for eleven OECD countries. Section 5 concludes.

II. Cointegration and the Term Structure

The expectations theory of the term structure posits that the long-term interest rate should be the average of current and expected future short-term interest rates. Campbell and Shiller (1991) noted that this hypothesis implies that a maturity specific multiple of the term spread predicts future changes in the long-term bond yield. Thus, the expectations theory of the term structure suggests that the current interest rate spread between long-term and short-term interest rates is an optimal predictor of future changes in long-term interest rates. According to this hypothesis, the market's expectations about bond yields are reflected in the slope of the term structure with a one-to-one relation. According to Campbell and Shiller (1987), non-stationary long-term and short-term interest rates should be cointegrated with a cointegrating vector of (1,-1). Furthermore, Campbell and Shiller (1987) show that the spread between the long-term and short-term interest rate should help to predict future changes in short-term interest rates (i.e. that the long-term interest rate is weakly exogenous in a vector error-correction model).

The empirical evidence relating to the expectations hypothesis is far from conclusive. With respect to the U.S., Campbell and Shiller (1991) found that the spread coefficient between the long-term and short-term interest rate is often significantly different from unity, rejecting the expectations hypothesis. In contrast, Hall, Anderson, and Granger (1992) find favorable evidence for the expectations hypothesis for the short end of the government security maturity spectrum. Campbell and Shiller (1991, p. 505) note that "…the slope of the term structure almost always gives a forecast in the wrong direction for the short-term change in the yield on the longer bond, but gives a forecast in the right direction for long-term changes in short rates." Hardouvelis (1994) finds evidence against the expectations hypothesis for the U.S., while finding more favorable evidence in favor of the hypothesis for the other G7 countries in his sample. Wallace and Warner (1996) reject the expectations hypothesis while Dotsey and Otrok (1995) find evidence in support of it, although they note that the results are sample specific. (Cuthbertson, 1996, finds weaker evidence in favor of the expectations theory at the short end of the maturity spectrum). Using zero-coupon bond yields, Engsted and Tanggaard (1994) find that they cannot reject the expectations hypothesis for Denmark. Mankiw (1986) reports results for Germany, which indicate that the spread between long-term and short-term interest rates predict the subsequent change in the long-term interest rate but in the wrong direction. Hardouvelis (1994) argues that the use of instrumental variables can reverse this negative correlation between the spread and the long-term interest rate. He finds support for the expectations hypothesis for France and Italy. Siklos and Wohar (1997) find evidence of cointegration across the term structure consistent with the expectations hypothesis for a number of industrialized countries.

Among the possible reasons for conflicting support for the expectations hypothesis may be the existence of structural changes or nonlinearities in this relationship that may affect agents' expectations. Numerous factors have been suggested to induce nonlinear behavior in the spread between long-term and short-term interest rates including changes in the manner in which monetary policy is conducted (Driffill et al., 1997) and the exchange rate (Dillen, 1997). Haug and Siklos (2005) extend the linear error correction model of the term structure of interest rates by allowing for nonlinear adjustment of the error-correction term and allow for nonlinearities in the short-run dynamics. They employ an exponential smooth transition autoregressive (ESTAR) model and undertake a careful specification of the transition variable that triggers a smooth transition from one regime to the next, based on economic explanations of nonlinearities. Using a sample of monthly data covering the period 1960-1998 they examine the term spread for Canada, Germany, Sweden, Switzerland, the UK, and the US. They find evidence of nonlinear adjustment in the error correction term all six countries. They offer a number of reasons to expect nonlinearity in the equilibrium adjustment process between long-term and short-term interest rates. These explanations include, differences in transaction costs across different maturities of bonds as well as over time, time varying risk premia, the sluggish adjustment of inflationary expectations to shocks, interest rate smoothing, differences in volatility between short-term versus long-term interest rates, and the adoption of inflation targets. All of these factors may upset the likelihood of a linear adjustment in the spread between long-term and short-term interest rates.

While some studies have found long-run evidence to support the expectations hypothesis (using cointegration techniques) there is much less support in the short-run. Two main explanations for the failure of the short-run tests have been put forth. The first is that long-term interest rates not only contain information about future short-term rates, but also about risk (or term) premia (see e.g. Fama, 1984; Mankiw, 1986; Tzavalis and Wickens, 1997, 1998). The second explanation is the over-reaction to future short-term rates (see e.g. Mankiw, 1986, Campbell and Shiller, 1991; and Hordouvelis, 1988, 1994)

III. Threshold Cointegration Models and Cointegration Tests

A. THRESHOLD COINTEGRATION MODELS

Threshold cointegration is a concept introduced by Balke and Fomby (1997), and is a natural extension of the original model of Engle and Granger (1987). Let z_t be the equilibrium error from a long-run regression of y_t on x_t. The equilibrium TAR (EQ-TAR) threshold cointegration model assumes that outside of a central band, $[\theta, -\theta]$, z_t is a stationary autoregressive process which reverts to its long-run mean (zero if a constant is included in the long-run model). An alternative to the EQ-TAR model is the BAND-TAR model, which assumes that outside the central band, $[\theta, -\theta]$, z_t is a stationary autoregressive process that reverts to the edge of the band. The EQ-TAR model can be written:

$$\begin{aligned} z_t &= \rho z_{t-1} + \varepsilon_t \text{if } z_{t-1} > \theta \qquad 0 \leq \rho < 1 \\ &= z_{t-1} + \varepsilon_t \text{if } \left| z_{t-1} \right| \leq \theta \\ &= \rho z_{t-1} + \varepsilon_t \text{ if } z_{t-1} < -\theta \end{aligned} \tag{1}$$

The BAND-TAR model can be written:

$$\begin{aligned} z_t &= \theta + \rho(z_{t-1} - \theta) + \varepsilon_t \text{if } z_{t-1} > \theta \\ &= z_{t-1} + \varepsilon_t \text{ if } \left| z_{t-1} \right| \leq \theta \\ &= -\theta + \rho(z_{t-1} + \theta) + \varepsilon_t \text{ if } z_{t-1} < \theta \end{aligned} \tag{2}$$

For the data used in this paper we experimented with both BAND-TAR and EQ-TAR specifications and found very similar results for both specifications. Hereafter, we restrict our attention to the EQ-TAR specification.

The EQ-TAR model given by equation (2) is a *symmetric* mean reversion threshold model: the degree of mean reversion is the same regardless of whether it is the upper threshold that is exceeded or whether it is the lower threshold that is exceeded. In the empirical analysis below, we employ *asymmetric* mean reversion EQ-TAR models for z_t. These models do not restrict the degree of mean reversion to be equal but allow for different degrees of mean reversion depending on whether the equilibrium error is above the upper threshold or below the lower threshold. An asymmetric mean reversion EQ-TAR model for the equilibrium error, z_t, is given as:

$$\begin{aligned} z_t &= \rho_1 z_{t-1} + \varepsilon_t \text{ if } z_{t-1} > \theta \\ &= z_{t-1} + \varepsilon_t \text{ if } \left|z_{t-1}\right| \leq \theta \\ &= \rho_2 z_{t-1} + \varepsilon_t \text{ if } z_{t-1} < -\theta \end{aligned} \tag{3}$$

If $\rho_1 \neq \rho_2$ the degree of mean reversion beyond the upper and lower thresholds is different. In addition to asymmetric mean reversion it is also possible to allow for *asymmetric thresholds*. An asymmetric mean reversion, asymmetric threshold EQ-TAR (ASEQ-TAR) model is written as:

$$\begin{aligned} z_t &= \rho_1 z_{t-1} + \varepsilon_t \text{ if } z_{t-1} > \theta_1 \\ &= z_{t-1} + \varepsilon_t \text{ if } \theta_2 \leq z_{t-1} \leq \theta_1 \\ &= \rho_2 z_{t-1} + \varepsilon_t \text{ if } z_{t-1} < \theta_2 \end{aligned} \tag{4}$$

where θ_2 is the lower threshold value and θ_1 is the upper threshold value.

B. COINTEGRATION TESTS FROM THRESHOLD MODELS

In this paper we build on the work of Balke and Fomby (1997) and Enders and Siklos (2001). The simulated critical values of Enders and Siklos (2001) are for tests of the null hypothesis of no cointegration allowing for asymmetric adjustment under the alternative hypothesis. Our tests are similar to those in Enders and Siklos (2001) since they are also tests of the null hypothesis of no cointegration, but they are different since under the alternative hypothesis we allow for threshold cointegration as defined by Balke and Fomby (1997). While Balke and Fomby (1997) illustrate that conventional methods appear to perform reasonably well in the presence of threshold cointegration, their experiments do reveal some loss of power and size distortion. It is entirely possible that one may fail to reject the null hypothesis of no cointegration due to the fact that conventional tests are mis-specified under the alternative hypothesis if cointegration is of the threshold variety.

Enders and Granger (1998) propose both *t*- and *F*-statistics for testing the null hypothesis of a unit root allowing for asymmetric adjustment under the alternative hypothesis. We apply the same types of test to the problem of testing the null hypothesis of no cointegration allowing for threshold cointegration under the alternative hypothesis. We employ a generalized version of the ASEQ-TAR model (4) for the equilibrium error z_t :

$$\begin{aligned}\Delta z_t &= \phi_1 z_{t-1} + \sum_{i=1}^{k} \gamma_i \Delta z_{t-i} + \varepsilon_t \text{ if } z_{t-1} > \theta_1 \\ &= \sum_{i=1}^{k} \gamma_i \Delta z_{t-i} + \varepsilon_t \text{ if } \theta_2 \le z_{t-1} \le \theta_1 \\ &= \phi_2 z_{t-1} + \sum_{i=1}^{k} \gamma_i \Delta z_{t-i} \varepsilon_t \text{ if } z_{t-1} < \theta_2\end{aligned} \tag{5}$$

where, $\phi_1 = \rho_1 - 1$ (interpreted as the speed of adjustment to restore long-run equilibrium above the upper threshold value), $\phi_2 = \rho_2 - 1$ (interpreted as the speed of adjustment to restore long-run equilibrium below the lower threshold value), and $\varepsilon_t \sim NID(0, \sigma^2)$.[1] One way of testing the null hypothesis of no cointegration which allows for threshold cointegration under the alternative hypothesis is to use the *F*-statistic for testing $H_0\colon \phi_1 = \phi_2 = 0$. This test is analogous to the Φ tests of the null hypothesis of a unit root developed by Enders and Granger (1998), and extended by Enders and Siklos (2001). We will refer to the *F*-statistic for testing the null hypothesis $H_0\colon \phi_1 = \phi_2 = 0$ in (5) as $F_{ASEQ\text{-}TAR}$.

Note that if either $\phi_1 \neq 0$ or $\phi_2 \neq 0$ in (5), then z_t does not contain a unit root. Therefore, one could use the most significant of the *t*-statistics on $\hat{\phi}_1$ and $\hat{\phi}_2$ as a test of the null hypothesis of no cointegration between y_t and x_t. This test will be referred to as a *t-Max* test (this test is analogous to the *t-Max* unit root tests proposed by Enders and Granger, 1998, extended by Enders and Siklos, 2001). We will refer to the *t*-statistics on $\hat{\phi}_1$ and $\hat{\phi}_2$ in the estimated versions of (5) as $t_{a(ASEQ\text{–}TAR)}$, $t_{b(ASEQ\text{–}TAR)}$. Thus, for each model the *t-Max* statistic is the minimum of $t_{a\,(.)}$, $t_{b\,(.)}$ (the most negative, and thus the test statistic that gives maximum support to the alternative hypothesis). We will refer to the *t-Max* statistics from each model individually as $t_{Max\,(.)}$ where (.) defines the model, e.g. $t_{Max\,(ASEQ\text{–}TAR)}$, and collectively as "*t-Max* tests".

The problem of unidentified thresholds under the null hypothesis of linearity has already been commented on, leading Enders and Granger (1998) and Enders and Siklos (2001) to rely on Monte Carlo simulation to obtain critical values for their tests. It is only recently that asymptotic theory on testing for a unit allowing for threshold autoregression has been formalised (see Caner and Hansen, 2001). This has not been extended to cases in which the null hypothesis of no cointegration is tested allowing for Balke and Fomby–type threshold cointegration under the alternative. Therefore in this paper, as in Enders and Granger (1998) and Enders and Siklos (2001), we employ Monte-Carlo simulation to derive critical values for our *t-Max* and *F*-tests and do not attempt to derive limiting distributions.

In order to evaluate how robust our critical values for these tests are to the assumed model under the null hypothesis, we simulate critical values assuming various models under the null hypothesis. The first null model we assume is that the long-term rate y_t and the short-term rate x_t , are generated as independent random walks. To simulate appropriate critical values we generated 10,000 independent random walks:

$$y_t = y_{t-1} + \varepsilon_t \tag{6}$$

$$x_t = x_{t-1} + \eta_t \tag{7}$$

where $\varepsilon_t \sim NID\ (0,1)$ and $\eta_t \sim NID\ (0,1)$, and for each of these random walks estimated a regression of y_t on x_t by OLS:

$$y_t = \beta x_t + z_t \tag{8}$$

For each of these 10,000 regressions we saved the fitted residuals $\hat{z}_t$, estimated the TAR model (5) by conditional OLS (CLS) for these residuals (setting k = 0), and calculated the relevant *t-Max* and *F*-statistics.[2] The 10%, 5% and 1% critical values determined from this procedure are given in Table 1.

TABLE 1–Simulated Critical Values for Threshold Cointegration Models

t-Max test critical values			
	10%	5%	1%
$t_{Max\ (ASEQ\text{–}TAR)}$ $k = 0$	-2.642	-2.896	-3.384
$t_{Max\ (ASEQ\text{–}TAR)}$ $k = 1$	-2.675	-2.917	-3.402
$t_{Max\ (ASEQ\text{–}TAR)}$ $k = 4$	-2.728	-2.975	-3.455
F-test critical values			
$F_{ASEQ\text{–}TAR}$ $k = 0$	4.921	5.847	8.097
$F_{ASEQ\text{–}TAR}$ $k = 1$	5.050	6.006	7.840
$F_{ASEQ\text{–}TAR}$ $k = 4$	5.169	6.135	8.167

Notes: All critical values were simulated for the empirical sample size 300, employing 10,000 replications in the Monte Carlo procedure and using the pseudo-random number generator in GAUSS for Windows version 3.2. k is the order of the additional lags of Δy_t and Δx_t included when simulating critical values.

In the second and third null models that we considered, the long-term rate y_t and the short-term rate x_t , are assumed to be independent I(1) processes, but with significant higher-order lags. In particular, for the second null model we assume that y_t and x_t are generated by the following equations:

$$\Delta y_t = .30\Delta y_{t-1} + \varepsilon_t \tag{9}$$

$$\Delta x_t = .30\Delta x_{t-1} + \eta_t \tag{10}$$

where $\varepsilon_t \sim NID\,(0,1)$ and $\eta_t \sim NID\,(0,1)$. Thus, we assume that under the null hypothesis of no cointegration the first and second lag of y_t and x_t in levels are significant. The parameter value in (9) and (10) of .30 was chosen on the basis of evidence from conventional linear models for the data considered below. For the third null model the Monte Carlo procedure described above was repeated, but with y_t and x_t generated by the following equations:

$$\Delta y_t = .30\Delta y_{t-1} + .15\Delta y_{t-4} + \varepsilon_t \tag{11}$$

$$\Delta x_t = .30\Delta x_{t-1} + .15\Delta y_{t-4} + \eta_t \tag{12}$$

where $\varepsilon_t \sim NID(0,1)$ and $\eta_t \sim \mathrm{NID}(0,1)$. So in this case we assume that under the null hypothesis of no cointegration that the fourth and fifth lag of y_t and x_t in levels, in addition to the first and second, lag are significant. Again, the parameter values (.30 and .15) were chosen on the basis of evidence from conventional linear models for the data considered below.

IV. Results

A. COINTEGRATION TESTS

The data for our analysis was collected from the IFS database on International Financial Statistics. It is monthly data for the period 1973:4 –1997:11. The short-term rate is the three-month Treasury bill rate, and the long-term rate is the 10-year Government Bond rate.

In Table 2 the results from the regressions of the long-term rate on the short-term rate are given. The results of the Engle-Granger test of the null hypothesis of no cointegration using the long-term and short-term interest rate is given in Table 2 (the ADF version), denoted t_{EG} , along with the ASEQ-TAR estimated parameters $\hat{\phi}_1$ and $\hat{\phi}_2$, the *t-Max* tests (below $\hat{\phi}_1$ and $\hat{\phi}_2$ in the table), the *F*-tests of the null of no cointegration derived from the ASEQ-TAR model, the estimated slope coefficient from the long-run regression of long-term interest rate on the short-term interest rate, denoted $\hat{\beta}$, and the associated *t*-statistic, $t_{\hat{\beta}}$. Also given in this table are the estimated threshold values $\hat{\theta}$.

TABLE 2–EG and ASEQ-TAR Tests of the Null Hypothesis of No Cointegration

	$\hat{\beta}$	$t_{\hat{\beta}}$	t_{EG}	$\hat{\phi}_1$ $t_{a(ASEQ-TAR)}$	$\hat{\phi}_2$ $t_{b(ASEQ-TAR)}$	$F_{ASEQ-TAR}$	$\lvert\hat{\theta}_1\rvert$	$\lvert\hat{\theta}_2\rvert$
Belgium	.726	34.409	-2.985	**-.098** **-2.945***	**-.100** **-3.029****	**8.450*****	.83	.68
Canada	.553	32.209	-2.928	-.075 -2.247	-.088 -2.273	4.746	.34	.36
France	.759	31.424	-3.320^b	**-.088** **-2.829***	-.063 -2.555	**6.925****	.90	.15
Germany	.452	20.948	-2.659	**-.095** **-3.197****	-.044 -1.263	**5.745***	.52	.43
Italy	.347	12.103	-2.175	-.045 -2.483	-.036 -1.609	4.245	1.380	1.180
Japan	.664	35.985	-3.061^a	-.077 -2.443	**-.129** **-3.459*****	**8.485*****	.92	.67
Netherlands	.371	15.624	-2.740	-.072 -2.012	-.103 -2.685	**5.361***	.81	.79
Norway	.631	24.252	-3.722^b	-.160 -2.350	**-.195** **-3.437****	**7.670****	.50	.48
Sweden	.432	18.774	-3.341^b	**-.127** **-2.882***	-.097 -2.687	**7.236****	.78	.25
UK	.514	13.834	-1.675	-.021 -1.537	-.027 -1.347	2.053	1.47	1.64
US	.700	29.041	-2.982	-.054 -1.946	**-.099** **-3.085****	**6.461****	.35	1.00

Notes: *, **, *** denote rejections at the 10%, 5% and 1% significance levels respectively, employment the critical values in Table 1 simulated assuming $k = 4$ (where $t_{a(ASEQ-TAR)}$ and $t_{b(ASEQ-TAR)}$ are both significant, the *t-Max* statistic is the most negative of these). Superscripts a and b denote a rejection of the null hypothesis of no cointegration from the Engle-Granger (ADF) test at the 10% and 5% significance levels using the critical values given in Engle and Yoo (1987).

In Table 2 and in all subsequent tables we set $k = 4$ in the tests employed (k being the number of lagged difference terms included when carrying out the tests). This choice was made following a preliminary empirical analysis in which all models were estimated setting $k = 6$ and then deleting insignificant lags. In nearly all cases $k = 4$ was chosen. While in some cases a value for $k < 4$ would have been sufficient, we felt it important to be generous in the lag specification to ensure that rejections were not being found from our tests due to an under-parameterised model.

The first point to note is that for all countries $0 \leq \hat{\beta} < 1$, and that for six of the eleven countries $\hat{\beta} > 0.5$. Thus, the estimated cointegrating vector appears to take on a sensible value for most countries, although it is below unity. Note also that for all countries this estimated parameter is highly significant. Consider next the results from the Engle-Granger test of the null hypothesis of no cointegration. While for some countries the null hypothesis of no cointegration is rejected, for many countries this is not the case. For example, for Germany and the U.S. we are unable to reject the null of no cointegration from the Engle-Granger test even at the 10% level of significance. Overall the Engle-Granger test yields a rejection of the null of no cointegration for only four of the eleven countries in our sample; France, Japan, Norway, and Sweden.

Comparing the calculated *t-Max* tests and *F*-tests derived from our ASEQ-TAR models with the appropriate critical values in Table 1 reveals evidence against the null hypothesis of no cointegration for eight out of the eleven countries. Compared to the results from the conventional Engle-Granger procedure, we now reveal an additional four rejections of the null hypothesis of no cointegration. Note in particular the results for Germany and the U.S. The Engle-Granger test fails to find evidence of cointegration for these countries, but from the *t-Max* tests and *F*-tests we are able to reject the null hypothesis of no cointegration at the 5% level of significance (interestingly there is no evidence against the null hypothesis of no cointegration for the U.K.). To clarify, consider the result for the U.S. as an example. For this country the estimated upper threshold is .35 and the estimated lower threshold is -1. The equilibrium error for the U.S. is $\hat{z}_t = y_t - .700x_t$. Thus, when $y_t > .700x_t + .35$ the long-term rate is in the upper-regime and has an AR(1) coefficient of 1 – .054 . Conversely, when the long-term rate is less than .700 multiplied by the short-term rate minus 1 percentage point, it is in the lower regime and has an AR(1) coefficient of 1 – .099. The evidence against the null hypothesis of no cointegration is particularly strong for the U.S., which is entirely what one would expect given the evidence from previous studies of nonlinearity in the U.S. term structure relationship.

B. SHORT-RUN ADJUSTMENT

As with linear cointegration, if two variables y_t and x_t are in a threshold cointegrating relationship, error correction models exist. In particular we can write a threshold error correction model as:

$$\Delta y_t = \gamma_1^i z_{t-1} + \nu_{1t} \tag{13}$$

$$\Delta x_t = \gamma_2^i z_{t-1} + \nu_{2t} \tag{14}$$

where $i = 1,2$ describes whether the data falls above the upper-threshold regime ($i = 1$) or below the lower-threshold regime ($i = 2$) respectively. The parameters γ_1^i , γ_2^i determine the short-run adjustment of y_t and x_t to deviations from the equilibrium relationships in the upper or lower regimes respectfully.

Alternatively we can use matrix notation to write the threshold VECM (adding one additional lag of Δy_t and Δx_t) as:

$$\begin{bmatrix} \Delta y_t \\ \Delta x_t \end{bmatrix} = \begin{bmatrix} \gamma_1^i \\ \gamma_2^i \end{bmatrix} z_{t-1} + \begin{bmatrix} \tau_{11} & \tau_{12} \\ \tau_{21} & \tau_{22} \end{bmatrix} \begin{bmatrix} \Delta y_{t-1} \\ \Delta x_{t-1} \end{bmatrix} + \begin{bmatrix} \nu_{1t} \\ \nu_{2t} \end{bmatrix} \tag{15}$$

Note that here we only allow the speed of adjustment parameters to be affected by the threshold. It is also of course possible to allow the parameters on the additional dynamic terms Δy_{t-1} and Δx_{t-1} to be regime dependent, although here experimentation showed that restricting the additional dynamic terms was acceptable for this data set. In practice we included four additional lags of Δy_t and Δx_t in our threshold VECM models and estimated the VECM using equation-by-equation CLS.[3]

The results from the estimation of threshold VECMs for the eleven OECD countries in our sample are given in Table 3 (we refer to these VECMs as asymmetric threshold VECMs (ASYM-VECM)). The superscript of 1 and 2 designates the upper and lower regime respectively, while the subscript of 1 denotes the long-term rate as the dependent variable and the subscript 2 denotes the short-term rate as the dependent variable. Note that the ASEQ-TAR models from which the thresholds are estimated (they are imposed in the VECMs), nest the symmetric threshold

VECM models (and for that matter the linear models).

TABLE 3–ASYM-VECM Results

Dependent variable	Long-term rate		Short-term rate	
	$\hat{\gamma}_1^1$	$\hat{\gamma}_1^2$	$\hat{\gamma}_2^1$	$\hat{\gamma}_2^2$
Belgium	**-.117** **-4.161**	-.004 -.148	-.022 -.527	**.125** **3.097**
Canada	**-.141** **-2.923**	-.056 -1.136	-.061 -.956	**.131** **1.715**
France	-.035 -1.243	-.005 -.221	.067 1.340	.071 1.667
Germany	**-.046** **-1.983**	-.032 -1.111	.073 1.107	.21 .265
Italy	.003 .183	-.006 -.317	**.108** **2.876**	**.091** **1.955**
Japan	-.018 -.583	-.066 -.317	.043 1.191	**.143** **3.121**
Netherlands	-.031 -1.214	-.023 -.818	.031 .292	**.295** **2.565**
Norway	**-.077** **-2.610**	**.040** **1.835**	.088 .588	**.396** **3.607**
Sweden	**-.125** **-3.520**	.008 .267	-.010 -.083	**.237** **2.352**
UK	-.020 -1.041	-.015 -.566	.001 .036	.027 .707
US	**-.109** **-3.350**	.015 .427	-.068 -1.229	**.159** **2.681**

Notes: The superscript of 1 and 2 designates the upper and lower regime respectively, while the subscript of 1 denotes the long-term rate as the dependent variable and the subscript 2 denotes the short-term rate as the dependent variable. The numbers in the top of each cell are the estimated speed of adjustment parameters, the numbers in the bottom of each cell are the associated t-statistics. All models were estimated by OLS imposing the thresholds estimated by CLS reported in Table 2. The parameters in bold are those which are statistically significant at the 10% level.

From these models we find that for six countries the speed of adjustment parameters relating to the upper regime for the long-term rate equation, $\hat{\gamma}_1^1$ are statistically significant from zero, but only for one country (Norway) is the speed of adjustment parameter relating to the lower regime for the long-term rate equation significant. For the short-term rate we find the opposite. For eight countries the speed of adjustment parameter relating to the lower regime is significant, but the speed of adjustment parameter relating to the lower regime is only significant for one country. For nearly all countries in this sample $\left|\hat{\gamma}_1^1\right| > \left|\hat{\gamma}_1^2\right|$ and $\left|\hat{\gamma}_2^1\right| < \left|\hat{\gamma}_2^2\right|$. This indicates that for nearly all countries, the adjustment of the long-term rate to disequilibrium is fastest when the equilibrium error is above the upper threshold, and the adjustment of the short-term rate to disequilibrium is fastest when the equilibrium error is below the lower threshold.

V. Conclusions

This paper builds on the work of Balke and Fomby (1997) and Enders and Siklos (2001). The simulated critical values of Enders and Siklos (2001) are for tests of the null hypothesis of no cointegration allowing for asymmetric adjustment under the alternative hypothesis. Our tests are similar to those in Enders and Siklos (2001) in that they are tests of the null hypothesis of no cointegration, but they differ in that the alternative hypothesis is threshold cointegration as defined by Balke and Fomby (1997), rather than the Enders and Siklos (2001) alternative of stationary TAR adjustment around a linear attractor. Their modeling framework does not allow for the possibility of a I(1) central regime. We also allow for a more general case of an asymmetric TAR alternative.

As an empirical application of our tests we investigate the term structure relationship between short-term and long-term interest rates for eleven OECD countries. Using the conventional Engle and Granger approach to testing no cointegration we find that the null hypothesis can be rejected for four of the eleven countries in our sample. The threshold cointegration tests proposed in this paper reveal evidence against the same null for eight of the eleven countries. We investigate the short-run adjustment of the long-term and short-term interest rates to disequilibrium using threshold VECMs. For nearly all countries we find that the long-term rate adjusts fastest to disequilibrium when it is above the upper threshold, while for the short-term rate the opposite is true. The tests proposed in this paper may have useful applications to other empirical relationships which economic theory suggests should be cointegrating, but which are found not to be when standard testing procedures are used. Examples include the existence of cointegration between nominal exchange rates, domestic prices and foreign prices as suggested by the theory of purchasing power parity; and the relationship between stock prices and dividends, which present value theory suggests should be cointegrating, but which is typically found not to be when conventional unit root tests are used (see e.g. Campbell and Shiller, 1987).

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Endnotes

1. As in Enders and Granger (1998) we restrict the higher-order dynamics to be the same irrespective of whether the series is inside or outside the band. This considerably simplifies the estimation problem. Experimentation with the actual data used in this paper confirmed that this was a reasonable assumption to make.
2. See Chan (1993) for more details on the CLS estimation of TAR models. Chan proves that for the case of a single threshold, CLS provides a superconsistent estimate of the threshold value.
3. This number of lags was found to be appropriate following a preliminary analysis of the data with conventional linear VECMs.