Dynamic Product Reliability Management for a Firm with a Complacent Competitor vs. a Lockstep Competitor

Jannett Highfill and Michael McAsey*

Abstract. Consider a dynamic duopoly model where R&D spending is used to increase the reliability of a firm's product under two competitive scenarios: the home firm competes with a "complacent" foreign firm that does no R&D whatsoever or with a "lockstep" foreign firm that improves its product at exactly the same rate as the home firm. The paper suggests that a firm with a complacent competitor produces a more reliable product, does more R&D, and earns more profit than a firm with a lockstep competitor. On the other hand, sufficiently less R&D spending is required that profits per R&D dollar are greater in the lockstep scenario. Extensions of the basic model include changes in the planning period, introducing trade costs, and considering intermediate competitive scenarios. (D92, O3)

I. Introduction

Research and development strategies are key to growth for many firms. While R&D expenditures can affect the firm in many ways, the present paper focuses on the situation when R&D spending is used to increase the reliability of a firm's product. The second focus is on the firm's competitive environment; assuming that the "home" firm has a "foreign" competitor, two extreme competitive scenarios are imagined. In the first, the foreign firm is "complacent" in that it does no R&D during the given planning period, and thus the reliability of its product is unchanged. In the "lockstep" competitive scenario, the foreign firm improves its product at exactly the same rate as the home firm. In this second scenario it is not possible for the home firm to gain a quality advantage, no matter how much it spends on R&D.

The primary finding of the paper suggests that a firm with a complacent competitor produces a more reliable product, does more R&D, and earns more profit than a firm with a lockstep competitor. Technological competition, where each firm keeps pace with the improvements of the other, leads to less product reliability improvement

*Jannett Highfill, Department of Economics, Bradley University, Peoria, IL 61625 and Michael McAsey, Department of Mathematics, Bradley University, Peoria, IL 61625.

and smaller profits. On the other hand, sufficiently less R&D spending is required that profits per dollar of R&D spending are greater in the lockstep scenario.

Beyond the basic model, the paper considers various extensions. The first considers a shorter or longer planning period; the second introduces trade costs making the necessary changes in the theory; and the third imagines intermediate cases between the complacent and lockstep competitive scenarios. Finally, a Monte Carlo simulation is used to conduct a sensitivity analysis for all the other parameters. In all cases the qualitative results are confirmed; the home firm in the complacent competitor scenario has higher reliability improvement, R&D spending, and profits, compared to the lockstep scenario, but lower profits per R&D dollar.

The instantaneous structure in the model of the present paper has many formal similarities with the R&D/quality literature where quality is chosen first and then quantities are chosen in a Cournot-type game; see Neary and Leahy (2000), Jinji and Toshimitsu (2006), DeCourcy (2005), Haaland and Kind (2008, 2006), and Gretz, Highfill, and Scott (2009). The focus in these papers is on policy issues relating to R&D subsidies, which the present paper does not consider; on the other hand they do not introduce dynamic considerations.

As for the theoretical literature on dynamic R&D spending, such paths in general equilibrium models are not uncommon, see Engers and Mitchell (2006), Laing, Palivos, and Wang (2002), and Russo (2004). For theory of the firm models, Buck and Stadler (1992) have an early study of dynamic R&D paths in a cost-minimization model. Dockner, Feichtinger, and Mehlmann (1993) consider the dynamic R&D paths of (identical) firms in a race to innovation. While our R&D function is similar to theirs, they have considerably less structure on the other components of profit. Hoppe and Lehmann-Grube (2001) have a study of first mover versus second mover advantage in innovation, but again with a very simple profit function. Confessore and Mancuso (2002) have a two period discrete time model with a structure similar to that of the present paper, except that they consider spillovers which the present paper does not, and its firms are identical which does not allow for a consideration of our competitive scenarios. Joshi and Vonortas (2001) also have a discrete time model with some formal similarities to the present paper except they have a somewhat specialized combination of production cost and R&D spending. They do however consider nonidentical firms, and their main result is that firms will become more similar over time.

Of particular interest are Petit and Tolwinski (1999) and Petit, Sanna-Randaccio and Tolwinski (2000). There are considerable formal differences between these papers and the current paper (their demand function is static, R&D affect only the firm's costs, time is discrete), but their cumulative R&D spending paths are similar to our product reliability paths and they examine various competitive scenarios – although the latter are quite different from those of the present paper. Finally it should be mentioned that there is an extensive related empirical literature on firm-level R&D practices. For a recent example see Lokshin, Belderbos, and Carree (2008).

The paper is organized as follows. The basic model which characterizes the firms' game and constructs the home firm's optimal control problem is presented in section II. An example is also provided. Section III considers three extensions: differing planning periods, trade costs, and intermediate competitive structures. The latter two extensions will require slight revisions of the basic model. Finally, in section IV, a Monte Carlo analysis is used to test the sensitivity of the basic model to changes in the demand side and marginal cost parameters. Numerical methods are used in all cases. The numerical algorithm used is often referred to as the Forward-Backward Sweep Method and uses a Runge-Kutta order 4 differential equation routine to solve the first order equations (10) resulting from the optimal control problem. See Lenhart and Workman (2007) for a discussion of the method.

II. The Basic Model and an Example

Consider a "home" firm with a single global competitor. The home firm, also called firm 1, is located in country *A* while the foreign firm 2, is located in country *B*. Both firms sell in the same market, which could be variously thought of as a world market (see Haaland and Kind (2008) for a survey of such models), the market in one or the other of the countries where the firms are located (as in Herguera and Lutz (2003); Herguera, Kujal and Petrakis (2000)), or indeed a third country market (as in DeCourcy (2005). Of course, where the product is sold affects the geographical distribution of benefits, but it does not affect the decisions of the firms in the case where there are no transportation or other trade costs as is assumed in this section. The case of two markets with transportation costs is considered in section III.B. The firms' revenues depend on the structure of the demand-side of the model, which itself depends on product reliability.

II.A DEMAND-SIDE ASSUMPTIONS

Suppose there is a distribution of customer reservation prices for a perfect product. Reservation prices are assumed to be uniformly distributed on the interval ($W(t)$, $V(t)$). That is, $W(t)$ is the minimum reservation price for the product at time t and $V(t)$ is the maximum reservation price. Customers are indifferent between the products of the two firms when products are perfectly reliable (in which case, as seen below, the two firms will charge the same price). These distributional demand assumptions are similar to Herguera and Lutz (2003) and Gretz, Highfill, and Scott (2009); Haaland and Kind (2008, 2006) arrive at a similar (linear) derived demand function by assuming a quadratic utility function, while DeCourcy (2005), d'Aspremont and Jacquemin (1988), Brod and Shivakumar (1997), and Greenlee (2005) forthrightly assume linear demand with no income effects.

Our measure of product reliability is the probability that a product is judged by the customer to be of acceptable quality; this probability is denoted $X_1(t)$ for the home firm and $X_2(t)$ for the foreign firm. This notion of quality as product reliability (or the related concept of product failure) can be found in Daughety and Reinganum (1995), Gretz, Highfill, and Scott (2009), and Matthews and Moore (1987). Product failure imposes costs on the customer that are not reimbursed by the firm. This "cost to customers" of product failure is the parameter $K(t)$, so the "expected cost of product failure" for a customer purchasing from the i^{th} firm is $(1-X_i(t))K(t)$ since $(1-X_i(t))$ is the probability of product failure. Customers know the probability that any arbitrary unit will fail, but not whether the particular unit they purchase will fail. At any moment t, customers whose reservation price $v(t)$ satisfies the following condition will purchase the product:

$$v(t) \geq P_1(t) + (1-X_1(t))K(t) = P_2(t) + (1-X_2(t))K(t) \qquad (1)$$

where $P_i(t)$ is the purchase price of the product from firm i. Customers are risk neutral in the sense that their buying decisions are based on price and expected customer cost of product failure. The expression $P_i(t) + (1-$

$X_i(t))K(t)$ is conveniently called the "full quality price." While in general the firms' costs, qualities, and prices are not the same, for both firms to have positive sales it must be the case that the full quality price is the same for both firms. If this were not the case, customers would only buy from the firm with the lower full quality price.

Under these assumptions the market quantity demanded is, using (1)

$$Q_M(t) = N(t)\int_{P_i(t)+(1-X_i(t))K(t)}^{V(t)} \frac{1}{V(t)-W(t)}\,dv$$
$$= \frac{N(t)}{V(t)-W(t)}\left(V(t) - P_i(t) - (1-X_i(t))K(t)\right)$$

where $Q_M(t) = Q_1(t) + Q_2(t)$, $Q_i(t)$ is quantity demanded for firm i, and $N(t)$ is the potential market size. Solving for the (indirect) demand functions yields

$$P_i(t) = V(t) - (1-X_i(t))K(t) - \frac{V(t)-W(t)}{N(t)}Q_M(t) \quad .$$

As for the time paths of the demand functions, assume the demand parameters have an exponential growth rate e^{rt}; intuitively, this growth rate may be related to the inflation rate. Assume the potential market size grows exponentially at a rate of e^{st}; this growth rate may be related to the population growth rate. Specifically, suppose

$$V(t) \equiv V_0e^{rt}, W(t) \equiv W_0e^{rt}, K(t) \equiv K_0e^{rt}, N(t) \equiv N_0e^{st}$$

with $K_0 > 0$, $V_0 > 0$, $W_0 > 0$, $N_0 > 0$, $r > 0$, and $s > 0$. The indirect demand functions are thus

$$P_i(t) = e^{rt}\left(V_0 - (1-X_i(t))K_0 - \frac{V_0 - W_0}{N_0e^{st}}Q_M(t)\right) \quad . \tag{2}$$

Finally, it might be noted that it is possible to view $X_i(t)$ as an abstract quality index, where one is highest possible quality and zero is the lowest possible quality. Thinking of quality abstractly has the advantage of being applicable in many different situations; it has the disadvantage that in practice almost always some proxy for quality must be used because of measurability issues.

II.B RESEARCH & DEVELOPMENT AND PRODUCTION ASSUMPTIONS

Improvements in quality require research and development; the home firm expenditure on R&D at time t is $E_1(t) \geq 0$. Assume that such expenditure produces an improvement in the reliability of the home firm's product, but is subject to diminishing marginal returns. Specifically,

$$dX_1 / dt = k(1 - X_1(t))\sqrt{E_1(t)} \tag{3}$$

where $k > 0$, $0 \leq X_1(t) \leq 1$. (Note that X_1 is the state variable; the E_1 is the control.) The assumption of a quadratic relationship between quality improvement and R&D spending which is independent of the quantity produced is found in Brod and Shivakumar (1997), d'Aspremont and Jacquemin (1988), Greenlee (2005), Herguera and Lutz (2003), Haaland and Kind (2006, 2008) and Gretz, Highfill, and Scott (2009). But here the relationship is modified by the term $(1 - X_1(t))$ which implies that the closer the reliability is to one at a given time the less productive a given level of R&D expenditure will be. Recall that $X_1(t)$ is a probability and so is between zero and one, while the expenditure on R&D is typically rather large and certainly never less than one. Therefore the constant k needs to reduce $\sqrt{E_1(t)}$ by several orders of magnitude and is typically a small fraction. Finally, although for the sake of simplicity we refer to $E_1(t)$ as R&D expenditure, it is really the component of expenditure which varies with reliability. There would normally be many fixed-cost R&D expenditures.

As mentioned above, two different competitive scenarios are considered. In both we assume that the two firms have the same initial reliability: $X_1(0) = X_{10} = X_2(0) = X_{20} = X_0$. That is, neither firm has a competitive advantage in reliability at the beginning of the planning period. In the "complacent competitor" scenario the foreign firm does not change its product reliability during the planning period, that is, $X_2(t) \equiv X_0$. In the "lockstep competitor" scenario the foreign firm instantly copies any product reliability improvements of the home firm, that is, $X_2(t) = X_1(t)$.

As for the firms' other costs, each firm has a per unit manufacturing cost of $mc_i(t)$. As with the demand parameters, suppose marginal costs grow exponentially at the rate e^{rt} so that $mc_i(t) \equiv mc_{i0}e^{rt}$ with $mc_{i0} > 0$. It is assumed that the units of the product that fail are returned by the customer and replaced (or if repaired that the repair cost is the same as the replacement cost) so that the total manufacturing costs are

$$(mc_{i0}e^{rt} + (1 - X_i(t))mc_{i0}e^{rt})Q_i(t). \tag{4}$$

That is, the manufacturing cost of the original units is $mc_{i0}e^{rt}Q_i(t)$; the (expected) cost of replacing or repairing the defective units is $(1-X_i(t))mc_{i0}e^{rt}Q_i(t)$, where $(1 - X_i(t))Q_i(t)$ is the expected number of defective units.

It will be useful to define "margin," the difference between the selling price and the per unit production cost using (2) and (4):

$$\begin{aligned} margin_i(t) &= P_i(t) - (mc_{i0}e^{rt} + (1 - X_i(t))mc_{i0}e^{rt}) \\ &= e^{rt}\left(V_0 - (1 - X_i(t))K_0 - \frac{V_0 - W_0}{N_0 e^{st}} Q_M(t) - mc_{i0} - mc_{i0}(1 - X_i(t))\right) \\ &= e^{rt}\left(V_0 - mc_{i0} - (K_0 + mc_{i0})(1 - X_i(t)) - \frac{V_0 - W_0}{N_0 e^{st}} Q_M(t)\right). \end{aligned}$$

This definition will be used shortly in the construction of variable profits.

II.C FIRM BEHAVIOR

At each time t during the planning period $0 \le t \le T$, the firms play a two-stage game with the order of play as follows. With the competitive scenario fixed for all t, product reliability is chosen as explained in the previous section. Then each firm chooses its own quantity taking the other firm's quantity (and both firms' qualities) as given. Solutions are computed using generalized backward induction. Therefore, the quantity decisions are considered first.

The assumptions above imply that variable profits are
$VP_i(t) = margin_i(t)Q_i(t)$.
The first order conditions ($\partial VP_i(t) / \partial Q_i(t) = 0$) imply

$$Q_1(t) = e^{st}\frac{N_0}{3(V_0 - W_0)}\left(\text{cnst}_{10} + 2(K_0 + mc_{10})X_1(t) - (K_0 + mc_{20})X_2(t)\right) \quad (5)$$

where $\text{cnst}_{10} = V_0 - K_0 - 4mc_{10} + 2mc_{20}$ and

$$Q_2(t) = e^{st}\frac{N_0}{3(V_0 - W_0)}\left(\text{cnst}_{20} - (K_0 + mc_{10})X_1(t) + 2(K_0 + mc_{20})X_2(t)\right)$$

where $\text{cnst}_{20} = V_0 - K_0 + 2mc_{10} - 4mc_{20}$.

The second order conditions are

$$\frac{d^2VP_i(t)}{dQ_i(t)^2} = -2\frac{V(t) - W(t)}{N(t)} < 0$$

which will be satisfied for feasible parameter sets. The first order conditions can be used to show

$$VP_i(t) = \frac{V(t) - W(t)}{N(t)}Q_i(t)^2. \quad (6)$$

Using (5) and (6) and focusing on the home firm,

$$VP_1(t) = \frac{1}{9}\frac{N_0 e^{(s+r)t}}{V_0 - W_0}\left(\text{cnst}_{10} + 2(K_0 + mc_{10})X_1(t) - (K_0 + mc_{20})X_2(t)\right)^2.$$

The optimal control problem for firm 1 can now be stated. In both of the competitive scenarios, the algebraic form of the problem is the same: a quadratic expression in X representing the instantaneous variable profits minus the instantaneous expenditure on R&D. For notational convenience the firm 1 subscripts are suppressed. Choose $E(t)$ to maximize the integral:

$$\max_E \int_0^T \left\{e^{\sigma t}\left[\alpha_0 + \beta_0 X(t)\right]^2 - e^{-\rho t}E(t)\right\}dt \quad (7)$$

subject to

$$dX/dt = k(1 - X(t))\sqrt{E(t)},\ X(0) = X_0 \quad (8)$$

$$0 \le X \le 1 \quad (9)$$

where $\sigma = s + r - \rho$, and ρ is the discount rate.

The constants α_0 and β_0 depend on the competitive scenario. In the case of the complacent competitor where $X_2(t) \equiv X_0$, the constant α_0 becomes

$$\sqrt{\frac{N_0}{9(V_0 - W_0)}}(V_0 - K_0 - 4\mathrm{mc}_{10} + 2\mathrm{mc}_{20} - (K_0 + \mathrm{mc}_{20})X_0)$$

and β_0 is

$$2\sqrt{\frac{N_0}{9(V_0 - W_0)}}(K_0 + mc_{10}).$$

In the case of the lockstep competitor where $X_2(t) \equiv X_1(t)$, the constant α_0 is

$$\sqrt{\frac{N_0}{9(V_0 - W_0}}(V_0 - K_0 - 4\mathrm{mc}_{10} + 2\mathrm{mc}_{20})$$

and β_0 is

$$\sqrt{\frac{N_0}{9(V_0 - W_0)}}(2(K_0 + \mathrm{mc}_{10}) - (K_0 + \mathrm{mc}_{20})).$$

The Hamiltonian is

$$H = e^{\sigma t}\left[\alpha_0 + \beta_0 X(t)\right]^2 - e^{-\rho t}E(t) + p(t)k(1 - X(t))\sqrt{E(t)}.$$

The first order condition is $\frac{\partial H}{\partial E} = 0$ which gives $0 = -e^{-\rho t} + \frac{p(t)k(1 - X(t))}{2\sqrt{E(t)}}$. This equation can be solved for

$E(t) = \left(\frac{1}{2}p(t)k(1-X(t))\right)^2 e^{2\rho t}$. The first order condition can alternatively be solved for $\sqrt{E(t)}$ and substituted into the state equation: $\sqrt{E(t)} = \frac{1}{2}p(t)k(1-X(t))e^{\rho t}$ so that

$$dX/dt = k(1-X(t))\left(\frac{1}{2}p(t)k(1-X(t))e^{\rho t}\right)$$

$$= \frac{1}{2}p(t)k^2(1-X(t))^2 e^{\rho t}.$$

The costate equation is given by $dp/dt = -\frac{\partial H}{\partial X}$

$$dp/dt = -2e^{\sigma t}\beta_0\left[\alpha_0 + \beta_0 X(t)\right] + p(t)k\sqrt{E(t)}$$

$$= -2e^{\sigma t}\beta_0\left[\alpha_0 + \beta_0 X(t)\right] + p(t)k\left(\frac{1}{2}p(t)k(1-X(t))e^{pt}\right)$$

$$= -2e^{\sigma t}\beta_0\left[\alpha_0 + \beta_0 X(t)\right] + \frac{1}{2}p(t)^2k^2(1-X(t))e^{\sigma t}.$$

So we get two first order differential equations.

$$dp/dt = -2e^{\sigma t}\beta_0\left[\alpha_0 + \beta_0 X(t)\right] + \frac{1}{2}p(t)^2k^2(1-X(t))e^{\sigma t}, p(T) = 0$$

$$dX/dt = \frac{1}{2}p(t)k^2(1-X(t))^2 e^{\rho t}, X(0) = X_0.$$

II.D RELIABILITY, R&D EXPENDITURE, AND SHADOW PRICE PATHS: AN EXAMPLE

The first step in characterizing an optimal solution is to examine the optimal reliability, R&D, and shadow-price paths; to that end an example will be constructed; the parameters are in line with the static analysis of Gretz, Highfill, and Scott (2009) except, of course, for the changes required for a dynamic model; the parameter values are given in the appendix.

Consider first the reliability paths (the state variable) for the home firm.

As shown in Figure 1, for both competitive scenarios the firm improves its product more quickly at the beginning of the planning period compared to the end.

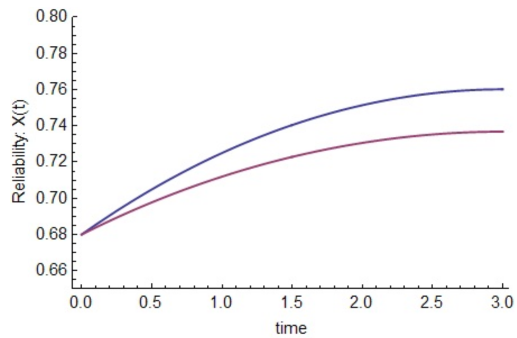

Figure 1. Reliability Paths:
Complacent Competitor (top curve) vs. Lockstep Competitor (bottom curve)

Considering now the differences between scenarios, a firm with a complacent competitor will produce a more reliable product at every moment than a firm with a lockstep competitor.

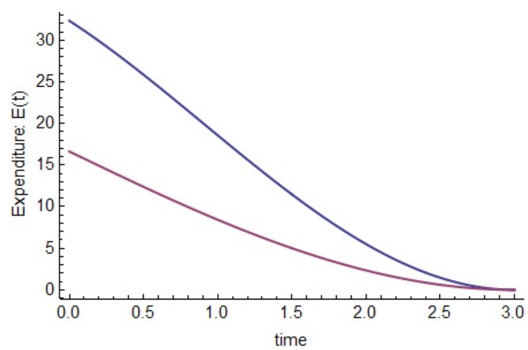

Figure 2. R&D Expenditure Paths:
Complacent Competitor (top curve) vs. Lockstep Competitor (bottom curve)

As for the control variable, R&D spending, as seen in Figure 2, the firm with a complacent competitor spends more than a firm with a lockstep competitor. Intuitively, the firm with complacent competitor spends more initially so that the reliability of its product is always better compared to a firm with a lockstep competitor even though actual expenditures become close for both scenarios toward the end of the planning period.

In order to examine the effectiveness of R&D spending over time it is useful to examine the shadow price paths. Intuitively, $p(t)$, the shadow price, measures the effort to increase the rate of improvement in the reliability of the product, i.e., the value to the firm of changes in the right-hand side of (3). For brevity, we will refer to the shadow price as the value of R&D spending.

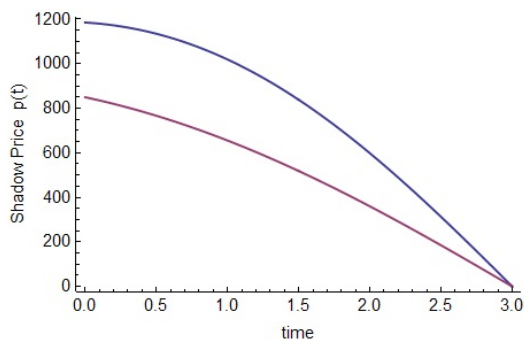

Figure 3. Shadow Price Paths:
Complacent Competitor (top curve) vs. Lockstep Competitor (bottom curve)

As shown in Figure 3, the value of R&D spending is higher for the complacent competitor scenario as compared to the lockstep scenario but both are declining as spending declines and reliability grows less quickly.

Given that the competitive scenarios lead to different reliability and shadow price paths, it is perhaps not surprising that other firm metrics can vary considerably between scenarios as well. Consider the results in Table 1.

TABLE 1–Home Firm Metrics for the Competitive Scenarios

	Complacent Competitor	Lockstep Competitor
Reliability Improvement	11.79%	8.35%
Profits	1,418	954
R&D Expenditure	39.27	18.36
Profits per R&D Dollar	36.11	51.94

"Reliability Improvement" is the percentage increase in the home firm's reliability during the planning period, i.e., $\frac{X_1(T) - X(0)}{X(0)}$. "Profits" are the total profits earned by the home firm during the planning period, i.e., the optimized value of (7). "R&D Expenditure" is total spending on R&D during the planning period, $\int_0^T E(t)dt$. Finally, "Profits per R&D Dollar" is the ratio of the two previous numbers.

Although the reliability improvement is smaller for the firm with a lockstep competitor compared to a firm with a complacent competitor as suggested by Figure 1, 11.79% improvement is almost once and a half as big as 8.35% improvement. Next, better product reliability translates into higher profits. The firm with a complacent competitor makes a more reliable product and earns higher profits than the firm with a lockstep competitor. Perhaps not surprisingly, better reliability is associated with greater R&D spending. But notice that the difference in R&D spending is large relative to the difference in profits, although profits are an order of magnitude larger than R&D spending. This explains the final result, that the return on the R&D dollar is much better for a firm with a lockstep competitor. Such a firm will spend much less on R&D, and only have a somewhat smaller profit.

III. Extensions of the Basic Analysis

The present section extends the analysis in three ways. The effect of differing choices of the planning period is explored first. Then the model is extended to incorporate trade costs; this requires the division of the single world market into two country markets. Finally, the case where the competitive environment is a mixture of the two competitive scenarios is examined.

III.A THE PLANNING PERIOD

The R&D being modeled here might be usefully thought of as producing "within generation" incremental improvements in the product. It might well be the case that one or both of the firms is doing R&D designed not to improve the current product but rather to create its replacement, i.e., the next generation of the product. The simplest way to think about this possibility is to suppose that the amount of R&D spending on the new generation product is an additional fixed cost, but that once the new generation product is introduced the market for the current generation completely dries up. In this case the appropriate planning period would be the number of years between generations.

The planning periods for R&D expenditures are taken here to be two to five years, $2 \le T \le 5$. Although many product generation examples could be cited, consider noise reducing headphones, with Bose as firm 1 and thinking of firm 2 as a composite of all other firms in the market. According to Wikipedia, Bose introduced its Aviation Headset Series I in 1989, the Aviation Headset Series II six years later in 1995, the QuietComfort Headphones five years after that in 2000, the QuietComfort Headphones 2 three years later in 2003, and the QuietComfort Headphones 3 three years later in 2006.

Tables 2 and 3 compare the results for firm 1 for such planning periods.

TABLE 2: Planning Period Effect for the Complacent Competitor Scenario

T	Reliability Improvement	Profits	R&D	$\frac{\text{Profits}}{\text{R\&D}}$
2	4.13%	630	6.09	103.52
3	11.79%	1,417	39.27	36.11
4	22.80%	3,118	155.41	20.06
5	30.82%	5,479	325.25	16.85

TABLE 3–Planning Period Effect for the Lockstep Competitor Scenario

T	Reliability Improvement	Profits	R&D	$\frac{\text{Profits}}{\text{R\&D}}$
2	3.56%	548	4.49	122.25
3	8.35%	954	18.36	51.94
4	14.86%	1,535	51.64	29.72
5	21.62%	2,320	108.56	21.37

The first thing to notice is that as might be expected the longer the planning period the greater the reliability improvement. Moving from a two year planning period to a three year planning period the reliability improvement more than doubles for both competitive scenarios. Although the incremental improvements from increasing the planning period by a year get smaller, the improvements are still substantial. For example, in the lockstep scenario a planning period of five years produces a reliability improvement (21.62%) half again as big as a planning period of four years (14.86%).

Comparing Tables 2 and 3, the qualitative results of Table 1 hold for all planning periods. The three firm metrics (reliability improvement, profits, and R&D) that were greater under the complacent competitor scenario remain greater for all planning periods. And same reversal occurs for profits per R&D dollar; it is higher for the lockstep scenario for all planning periods.

III.B THE TWO MARKET MODEL

As in the previous model firm 1 is located in country *A*, and firm 2 is located in country *B*. The primary distinction between this model and the one market model is that each country is also a market, and both firms sell in both markets. Using reasoning similar to that above, suppose reservation prices at any time *t* in country *A* are distributed uniformly on the interval $(W_A e^{rt}, V_A e^{rt})$ and for country *B* on the interval $(W_B e^{rt}, V_B e^{rt})$. The derivation of the indirect demand functions proceeds as before. The indirect demand function for firm 1 in country *A* is

$$P_{A1}(t) = e^{rt}\left(V_{A0} - (1 - X_1(t))K_{A0} - \frac{V_{A0} - W_{A0}}{N_{A0}e^{st}}(Q_{A1}(t) + Q_{A2}(t))\right)$$

The other indirect demand functions are similar.

This division of the world market into two separate country markets does not affect the R&D function for firm 1, i.e., equation (3). For the production costs, it is still assumed that each firm has a per unit manufacturing cost of $mc_i(t)$. However now there is also a per unit "trade cost" τ for goods that are exported: $mc_{A10} = mc_{10}$, $mc_{B10} = mc_{10} + \tau$, $mc_{A20} = mc_{20} + \tau$, $mc_{B20} = mc_{20}$. Trade costs are positive when firm 1 exports to country B and firm 2 exports to country A. The normal interpretation of trade costs is as transportation costs, but following Haaland and Kind (2008, p. 173) it could also be interpreted as "a synthetic measure of a wide range of barriers to trade including transport costs, costs of frontier formalities, and differing product standards."

The total manufacturing costs are the same as in equation (4), adjusting for the change in notation and interpretation of the marginal costs. For example, the production and replacement costs for the output that firm 1 sells to country *A* are

$$(mc_{A10}e^{rt} + (1 - X_1(t))mc_{A10}e^{rt})Q_{A1}(t).$$

The profit margins and variable profits are also similar to those above with the change in notation and interpretation. The profit margin for firm 1 sales in country *A* is

$$margin_{A1}(t) = e^{rt}\left(V_{A0} - mc_{A10} - (K_{A0} + mc_{A10})(1 - X_1(t)) - \frac{V_{A0} - W_{A0}}{e^{st}N_{A0}}Q_A(t)\right)$$

where $Q_A(t) = Q_{A1}(t) + Q_{A2}(t)$. The other margins are defined similarly. As in section II.A, variable profits are margin times quantity sold so that

$$VP_1(t) = VP_{A1}(t) + VP_{B1}(t) = margin_{A1}(t)Q_{A1}(t) + margin_{B1}(t)Q_{B1}(t)$$
$$VP_2(t) = VP_{A2}(t) + VP_{B2}(t) = margin_{A2}(t)Q_{A2}(t) + margin_{B2}(t)Q_{B2}(t)$$

The first order conditions imply

$$Q_{A1}(t) = e^{st}SC_{A0}\left(cnst_{A10} + 2(K_{A0} + mc_{A10})X_1(t) - (K_{A0} + mc_{A20})X_2(t)\right)$$

where $SC_{A0} = \dfrac{N_{A0}}{3(V_{A0} - W_{A0})}$ and $\text{cnst}_{A10} = V_{A0} - K_{A0} - 4mc_{A10} + 2mc_{A20}$,

$$Q_{A2}(t) = e^{st} SC_{A0}\left(\text{cnst}_{A20} - (K_{A0} + mc_{A10})X_1(t) + 2(K_{A0} + mc_{A20})X_2(t)\right)$$

where

$SC_{A0} = \dfrac{N_{A0}}{3(V_{A0} - W_{A0})}$ (as in $Q_{A1}(t)$) and $\text{cnst}_{A20} = V_{A0} - K_{A0} + 2mc_{A10} - 4mc_{A20}$,

$$Q_{B1}(t) = e^{st} SC_{B0}\left(\text{cnst}_{B10} + 2(K_{B0} + mc_{B10})X_1(t) - (K_{B0} + mc_{B20})X_2(t)\right)$$

where

$SC_{B0} = \dfrac{N_{B0}}{3(V_{B0} - W_{B0})}$ and $\text{cnst}_{B10} = V_{B0} - K_{B0} - 4mc_{B10} + 2mc_{B20}$, and

$$Q_{B2}(t) = e^{st} SC_{B0}\left(\text{cnst}_{B20} - (K_{B0} + mc_{B10})X_1(t) + 2(K_{B0} + mc_{B20})X_2(t)\right)$$

where

$SC_{B0} = \dfrac{N_{B0}}{3(V_{B0} - W_{B0})}$ (as in $Q_{B1}(t)$) and $\text{cnst}_{B20} = V_{B0} - K_{B0} + 2mc_{B10} - 4mc_{B20}$

The strategy for arriving at firm 1's objective function for the dynamic problem is now the same as in the one market model (see (6)) noting that

$$VP_1(t) = VP_{A1}(t) + VP_{B1}(t) = \frac{V_A(t) - W_A(t)}{N_A(t)} Q_{A1}(t)^2 + \frac{V_B(t) - W_B(t)}{N_B(t)} Q_{B1}(t)^2 .$$

The effect of trade costs on the various firm metrics under the two competitive scenarios is examined next. The example whose results are summarized in Table 1 is extended by allowing for positive trade costs, τ, and dividing the potential market size evenly between countries *A* and *B*. The relationship between the single market model and the two market model can be seen in the first row of Tables 4 and 5. When trade costs are zero the results here are the same as for the one market model.

TABLE 4–Trade Cost Effect for the Complacent Competitor Scenario

Trade Cost, τ	Reliability Improvement	Profits	R&D	Profits / R&D
0	11.79%	1,418	39.27	36.11
1	11.60%	1,385	37.77	36.67
2	11.41%	1,355	36.42	37.22
3	11.25%	1,328	35.19	37.74
4	11.10%	1,304	34.13	38.21
5	10.96%	1,283	33.17	38.66
6	10.84%	1,264	32.34	39.08
7	10.74%	1,248	31.64	39.45
8	10.65%	1,235	31.04	39.78
9	10.58%	1,225	30.59	40.04
10	10.53%	1,218	30.25	40.25

TABLE 5–Trade Cost Effect for the Lockstep Competitor Scenario

Trade Cost, τ	Reliability Improvement	Profits	R&D	Profits / R&D
0	8.35%	954	18.36	51.94
1	8.21%	933	17.66	52.83
2	8.08%	914	17.03	53.67
3	7.96%	897	16.48	54.45
4	7.85%	883	16.01	55.16
5	7.76%	870	15.60	55.80
6	7.68%	860	15.26	56.39
7	7.62%	852	14.99	56.82
8	7.57%	846	14.78	57.19
9	7.54%	841	14.64	57.46
10	7.52%	839	14.56	57.62

The effect of increasing trade costs is to reduce reliability improvement, R&D expenditure, and profits. In other words, trade liberalization – a decrease in trade costs – increases the firm's spending on R&D, which is consistent with Haaland and Kind (2008, p. 177), implying a relative improvement in reliability as well. Profits increase because the improvements in reliability more than pay for themselves. Comparing the competitive scenarios, the results are consistent with those in sections II.D and III.A. The complacent competitor scenario has more reliability improvement, profits, and R&D spending than the lockstep competitor scenario. Profits per R&D dollar, as suggested by previous results, are higher for the lockstep scenario.

Perhaps the most surprising result from the two market model concerns the effect of trade liberalization on profits per R&D dollar. In both competitive scenarios a reduction in trade costs is associated with lower profits per R&D dollar.

III.C INTERMEDIATE COMPETITIVE SCENARIOS

Returning now to the single market model, i.e., $\tau = 0$, suppose that the competitive environment is a mixture of the two competitive scenarios. That is, suppose

$$X_2(t) = (1 - \xi)X_0 + \xi X_1(t)$$

where the parameter ξ is between zero and one. That is, the home firm 1 knows that its competitor's reliability path will be a weighted average of the initial reliability (as in the complacent competitor scenario) and its own (as in the lockstep scenario). It may be useful again in this case to think of firm 2 as a composite of all other firms in the market. The previous scenarios become special extreme cases; the complacent competitor scenario is the case $\xi = 0$ and the lockstep scenario is when $\xi = 1$. In Table 6 these are the first and last rows respectively, which repeat the information in Table 1.

TABLE 6–Trade Cost Effect for the Lockstep Competitor Scenario

ξ	Reliability Improvement	Profits	R&D	$\frac{\text{Profits}}{\text{R\&D}}$
0	11.91%	1418	39.27	36.11
0.1	11.33%	1349	35.86	37.62
0.2	10.89%	1287	32.85	39.18
0.3	10.50%	1231	30.22	40.73
0.4	10.12%	1180	27.89	42.32
0.5	9.78%	1134	25.83	43.89
0.6	9.45%	1091	23.98	45.50
0.7	9.15%	1052	22.35	47.10
0.8	8.87%	1017	20.87	48.72
0.9	8.60%	984	19.55	50.33
1.0	8.35%	954	18.36	51.94

The qualitative firm results emphasized in sections II.D, III.A, and III.B occur here for the intermediate competitive scenarios: the closer the lockstep scenario comes to describing the competitive situation, the lower the reliability improvement, R&D, and profits, but the greater the profits per R&D dollar.

For a given value of the competitive scenario parameter ξ, the home firm knows what proportion of its reliability improvement will be copied by its foreign rival. Equivalently, it can compute the "lag" between the time it achieves a given level of reliability and when its rival achieves that same level of reliability. Suppose, for example, $\xi = .75$; as a manner of speaking we might say that the competitive scenario is three fourths lockstep and one fourth complacent competitor.

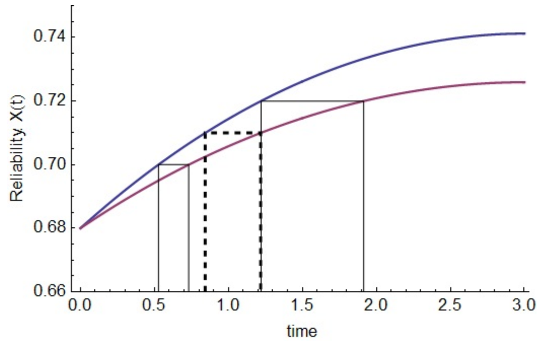

Figure 4. Reliability Improvement Lags:
Home Firm (top curve) vs. Foreign Firm (bottom curve)

In Figure 4, the home firm will achieve a 70% reliability level in slightly over a half a year while the foreign firm achieves a 70% reliability level in slightly under three fourths of a year. The home firm achieves a 72% reliability in about 1.25 years while the foreign firm does not attain a 72% reliability level until almost the second year. Notice two things: first, the time lags get longer as reliability increases, and second, there is a limited range of home firm reliabilities that the foreign firm will achieve during the planning period. For this example, reliability above about 72.6% will never be achieved by the foreign firm.

IV. Monte Carlo Analysis

While the previous sections have examined the robustness of the qualitative results of Table 1 with regard to planning period, trade costs, and the intermediate competitive scenarios, this section considers the effect of changing the demand side and marginal cost parameters assuming the original planning period and no trade costs, again comparing the extreme competitive scenarios. (See the Appendix for details of the simulation.) The results are reported in Tables 7 and 8. Reliability improvement, profits, and R&D spending are higher in the complacent competitor scenario but profits per R&D dollar are lower. These qualitative results hold for each parameter set that generated an interior solution to the optimization problem (7)-(9) and are consistent with the results reported Sections II and III.

TABLE 7–Simulation Results for Firm with a Complacent Competitor

	Mean	St. Dev.	Minimum	Maximum
Reliability Improvement	10.90%	10.43	1.31%	49.31%
Profits	2,174	1,611	81	7,426
R&D Expenditure	73.06	53.26	12.49	268.85
Profits per R&D Dollar	73.12	55.82	13.29	271.77

TABLE 8–Simulation Results for Firm with a Lockstep Competitor

	Mean	St. Dev.	Minimum	Maximum
Reliability Improvement	8.22%	7.35	1.25%	41.87%
Profits	1,737	1,368	63	6,902
R&D Expenditure	37.54	48.03	0.26	331.56
Profits per R&D Dollar	88.56	59.95	14.27	285.62

The analysis so far has assumed that the firm's goal is to simply maximize profits; a firm spends as many dollars on R&D as needed to maximize profits given the competitive structure they find themselves in. This would make sense in a world without constraints on the number of dollars available for R&D spending. But consider for a moment a situation where R&D dollars are scarce; in that case the firm might focus on profits per dollar spent on R&D, rather than just profits. (This "return on investment" approach is quite common in the investment literature, and not unknown in the R&D literature (Tassey, 2005)).

We have seen that profits per R&D dollar are higher in the lockstep competitor scenario compared to the complacent competitor scenario under every variation of the model and with all parameter sets. Consider a firm in a startup operation, not yet committed to a particular product line and which has limited funds for R&D. If such a firm has a choice between an industry with a complacent competitor and a lockstep one, it might choose the competitive situation with a lockstep competitor. Never winning (but never losing) the technology race is best for profits per R&D dollar.

In both competitive scenarios, from Figure 5, for parameter sets with a large reliability improvement, the firm must spend enough R&D dollars that profits per R&D dollar are rather low. On the other hand, small reliability improvements are associated with high profits per R&D dollar.

But notice that the relationship is nothing like linear; the tradeoff between reliability improvement and profits per R&D dollar falls very quickly for low values of reliability improvement, less quickly for intermediate values, and very slowly for high values.

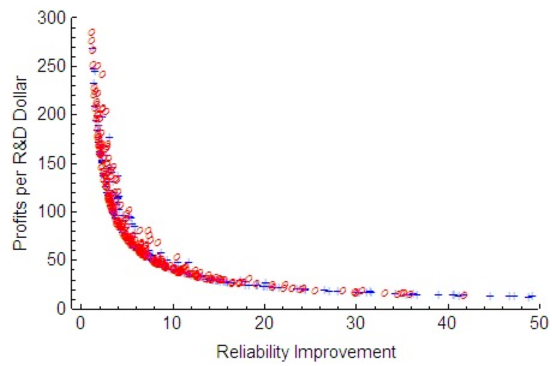

Figure 5. Reliability Improvement vs Profit/R&D: Complacent Competitor ("+") vs. Lockstep Competitor ("o")

V. Conclusion

This paper has considered the R&D path of a firm with a foreign competitor, focusing on the competitive situation between the two firms. Generally our results suggest that to the extent that the competitive scenario resembles the case of a complacent competitor, reliability improvement, R&D spending, and profits are greater compared to other competitive scenarios. On the other hand, when the competitive situation resembles that of a lockstep competitor which improves its product at exactly the same rate as the home firm, our results suggest that profits per dollar of R&D spending are higher. We find that in situations when a longer planning period is possible, more reliability improvement is achieved and more profits earned, but the profits per dollar of R&D spending decline. Trade liberalization has a similar effect, increasing reliability improvement, R&D spending, and profits, but decreasing the profits per dollar of R&D spending.

Appendix

The parameter set for the example of section II is given in Table A.1.

TABLE A.1 Example Parameters

Parameter	W_0	V_0	N_0	mc_{10}	mc_{20}	K_0
Value	100	200	60	72	72	140
Parameter	r	s	ρ	X_{10}	X_{20}	k
Value	0.025	0.025	0.025	0.68	0.68	0.03

The parameters sets for the Monte Carlo simulation are summarized in Table A.2. Each parameter set was constructed by choosing W_0, V_0, N_0, mc_{10}, mc_{20}, r, s, ρ, X_0 from a 60%-interval (±30%) around the corresponding parameter in the example of section II. For example, $W_0 = 100$ so for the Monte Carlo simulation, the corresponding parameter W_0 was chosen randomly from the interval [70, 130], that is, 100 ± 30%. The values of all the parameters listed above were chosen independently and randomly from their respective intervals. The parameter which was not varied was k, because, as suggested by Gretz, Highfill, and Scott (2009) it must be orders of magnitude smaller than the other parameters. Therefore $k = 0.03$ for all observations.

The selection of 60%-intervals was determined by two factors. First, for each parameter set, the resulting data needed to be certified as a solution. In particular the resulting prices needed to be in the interval [W_0, V_0] and the resulting quantities needed to be positive. Using the 60% intervals there were 215 observations that generated actual solutions out of 600 randomly generated parameter sets. Increasing the width of the randomization interval resulted in significant decreases in the number of usable data sets. The second factor in the choice of the size of the randomization interval is that the numerical algorithm that was used to produce solutions of the optimal control problem needed to converge. With significantly longer randomization intervals, the number of data sets that failed to produce convergent solutions increased significantly.

TABLE A.2–Descriptive Statistics for Parameters Yielding

Internal Solutions

Parameter	Mean	St. Dev.	Minimum	Maximum
W_0	92.10	13.44	70.33	127.18
V_0	220.88	26.85	141.53	259.84
N_0	61.34	9.70	42.03	77.75
mc_{10}	73.53	11.72	50.53	93.56
mc_{20}	73.17	11.66	50.48	93.51
r	0.0254	0.0042	0.0176	0.0324
s	0.0245	0.0044	0.0175	0.0324
ρ	0.0253	0.0044	0.0175	0.0325
X_0	0.7208	0.1086	0.4806	0.8838
K_0	134.52	24.26	98.44	181.92

References

Brod, Andrew, and Ram Shivakumar. 1997. "R&D Cooperation and the Joint Exploitation of R&D." *Canadian Journal of Economics*, 30(3):673-84.

Buck, Andrew J., and Manfred Stadler. 1992. "R&D Activity in a Dynamic Factor Demand Model: A Panel Data Analysis of Small and Medium Size German Firms." *Empirica*, 19(2):161-80.

Confessore, Giuseppe, and Paolo Mancuso. 2002. "A Dynamic Model of R&D Competition." *Research in Economics*, 56(4):365-80.

d'Aspremont, Claude, and Alexis Jacquemin. 1988. "Cooperative and Noncooperative R&D in Duopoly with Spillovers." *American Economic Review*, 78(5):1133-37.

Daughety, Andrew F., and Jennifer F. Reinganum. 1995. "Product Safety: Liability, R&D, and Signaling." *American Economic Review*, 85(5):1187-1206.

DeCourcy, Julie. 2005. "Cooperative R&D and Strategic Trade Policy." *Canadian Journal of Economics*, 38(2): 546-73.

Dockner, Engelbert J., Gustav Feichtinger, and Alexander Mehlmann. 1993. "Dynamic R&D Competition with Memory." *Journal of Evolutionary Economics*, 3(2):145-52.

Engers, Maxim, and Shannon K. Mitchell. 2006. "R&D Policy with Layers of Economic Integration." *European Economic Review*, 50(7):1791-1815.

Greenlee, Patrick. 2005. "Endogenous Formation of Competitive Research Sharing Joint Ventures." *Journal of Industrial Economics*, 53(3):355-91.

Gretz, Richard T., Jannett Highfill, and Robert C. Scott. 2009. "Strategic Research and Development Policy: Societal Objectives and the Corporate Welfare Argument." *Contemporary Economic Policy*, 27(1):28-45.

Haaland, Jan, and Hans Jarle Kind. 2006. "Cooperative and Non-cooperative R&D Policy in an Economic Union." *Review of World Economics/Weltwirtschaftliches Archiv*, 142(4):720-45.

Haaland, Jan I., and Hans Jarle Kind. 2008. "R&D Policies, Trade and Process Innovation." *Journal of International Economics*, 74(1):170-87.

Herguera, Inigo, Praveen Kujal, and Emmanuel Petrakis. 2000. "Quantity Restrictions and Endogenous Quality Choice." *International Journal of Industrial Organization*, 18(8):1259-77.

Herguera, Inigo, and Stefan H. Lutz. 2003. "The Effect of Subsidies to Product Innovation on International Competition." *Economics of Innovation and New Technology*, 12(5):465-80.

Hoppe, Heidrun C., and Ulrich Lehmann-Grube. 2001. "Second-Mover Advantages in Dynamic Quality Competition." *Journal of Economics and Management Strategy*, 10(3):419-33.

Jinji, Naoto, and Tsuyoshi Toshimitsu. 2006. "Optimal Policy for Product R&D with Endogenous Quality Ordering: Asymmetric Duopoly." *Australian Economic Papers* 45(2): 127-40.

Joshi, Sumit, and Nicholas S. Vonortas. 2001. "Convergence to Symmetry in Dynamic Strategic Models of R&D: The Undiscounted Case." *Journal of Economic Dynamics and Control*, 25(12):1881-97.

Laing, Derek, Theodore Palivos, and Ping Wang. 2002. "Product Diffusion and Pricing with Market Frictions." *Economic Theory*, 19(4):707-36.

Lenhart, Suzanne and John T. Workman. 2007. *Optimal Control Applied to Biological Models*. Boca Raton, FL: Chapman & Hall/CRC.

Lokshin, Boris, Rene Belderbos, and Martin Carree. 2008. "The Productivity Effects of Internal and External R&D: Evidence from a Dynamic Panel Data Model." *Oxford Bulletin of Economics and Statistics* 70(3):399-413.

Matthews, Steven, and John Moore. 1987. "Monopoly Provision of Quality and Warranties: An Exploration in the Theory of Multidimensional Screening." *Econometrica*, 55(2):441-67.

Neary, J. Peter, and Dermot Leahy. 2000. "Strategic Trade and Industrial Policy towards Dynamic Oligopolies." *Economic Journal*, 110(463): 484-508.

Petit, Maria-Luisa, Francesca Sanna-Randaccio, and Boleslaw Tolwinski. 2000. "Innovation and Foreign Investment in a Dynamic Oligopoly." *International Game Theory Review*, 2(1):1-28.

Petit, Maria Luisa, and Boleslaw Tolwinski. 1999. "R&D Cooperation or Competition?" *European Economic Review*, 43(1):185-208.

Russo, Benjamin. 2004. A Cost-Benefit Analysis of R&D Tax Incentives. *Canadian Journal of Economics*, 37(2):313-35.

Tassey, Gregory. 2005. "Underinvestment in Public Good Technologies." *Journal of Technology Transfer*, 30(1-2):89-113.